ICLP 2000
Rhodes - Greece
18-22 September 2000

AN APPROACH IN MODELLING OF LIGHTNING PROCESSES USING CELLULAR AUTOMATA

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Abstract: In the present work we proposed a new discrete stochastic model for computer simulation of the lightning process using Cellular Automata. Two different states of conductive structure were introduced that correspond to streamer and leader. The conductivity of the leader was supposed to be very high, while the conductivity of the streamers was supposed to be very low. The electric field potential was obtained by solving Laplace equation in the region outside the equipotential leader structure. If the energy released in some segment of streamer was larger than some critical value, the transformation of streamer to leader occurred. By applied this model for computer simulation, patterns of lightnings have been obtained.

Keywords: lightning, computer simulation, stochastic model, cellular automata.

1. INTRODUCTION

Lightning is a physical phenomenon with complex evolution. The description of this phenomenon is thus difficult, because many different and, sometimes, unknown factors (like humidity, air density, atmospheric ionization, etc.) should be taken into account. However, all these uncertainties of the lightning process can be considered as stochastic features of the model approach. In this sense, a new stochastic model was proposed for the simulation of the formation of lightning tree.

Stochastic models are widely used to simulate the propagation of a conductive phase before the breakdown in solid, gaseous and liquid dielectrics. One of the first model for computer simulation of the breakdown in dielectrics was the Niemeyer – Pietronero – Wiesmann (NPW) model [1] in which for the first time the probability of streamer growth was related to local electric field. In models, where the conductive tree was considered as equipotential one and Laplace equation was solved, the patterns of structures consisted rather of many small branches. However, this was not in good agreement either with the recordings of lightnings or with experimental observations. In work [2] the authors for the first time tried to solve this problem and used the special graphic post-processing to this purpose. They introduced the width of the branches as a measure of the channel luminosity and choose it proportional to the logarithm of the charge flow.

Actually the problem is in absence of complete charge relaxation along the branches of conductive structure. In another words, the process of gradual charge relaxation prevents electric field to enhance up to too high values. Thus, streamer tips propagate in the local field that greater but does not exceed several times the value of initial electric field. To take this into account the approximate approach was used in [3] to simulate breakdown in dielectrics. In this work the stochastic cellular automaton was used instead of exact calculations of electric field in the condition of absence of complete charge relaxation. To take into account the charge relaxation exactly it is necessary to solve the Poisson equation together with the equation of electric charge flow along the branches of finite conductivity [4,5].

Up to date computer simulations of streamer growth are based on the idea of space and time discretization. New linear segments of streamer channels join sequentially neighbor sites of some spatial lattice to the streamer-leader structure. So, the shape of the streamer-leader tree is represented by a connected graph consisting of conductive bonds.

It was proposed in [4] to separate all possible stochastic models of streamer growth into two groups. The first one includes the models in which only one bond is added in a time step (single-element models). The second group consists of models in which several bonds may be generated in each time step (multi-element models). For the first group the time step is equal to the delay time of the appearance of the first new bond. In the second ones the time step is constant and all the bonds that have time to arise less than the time step are accepted. The sequence of time intervals for each growth step calculated in a proper way according to certain rules is named as the “physical time”. From a physical point of view, in the single-element models, it is supposed that the growth of the first bond (streamer) suppresses the development of the others at current time step. On contrary in the multi-element models, the appearance of any bond does not influence on the development of the others.

2. CELLULAR AUTOMATA

In present work we develop the Niemeyer – Pietronero – Wiesmann approach to simulation of gas discharges [1] and propose a new stochastic model, taking into account the growth of the initial streamers and following transition several of them to leader. To this purpose we use cellular automata.

Cellular automata provide mathematical models for a wide variety of complex natural phenomena, from growth of patterns in biological systems to turbulence in fluids. Briefly a cellular automaton consist of:

- A lattice of cells. Each cell can be in one of a finite number of distinct states at each moment of time. This lattice may be two- or three-dimensional and of arbitrary size.
- Transformation rules from one state to another depending on only neighborhood defined for each cell.

In this work the cells have three states and the cellular automata are two-dimensional.

For the development of the model we use the well-known fact that the lightning consist of a sequence of streamer-leader phenomena. The channels of streamer are assumed to have low conductivity and luminosity and may transform to leader segments with high conductivity and luminosity after some short time. Taking this into account, the space between the cloud and the earth is divided into cells. Each cell occupied by dielectric can be in three states. The first state is initial one. It means that in this cell nothing has happened. The second state corresponds to the formation of the streamer in this cell. The third state denotes that the transition of the streamer state to the leader state has occurred.

Before the initiation of the lightning, all cells were in the initial state (S1). Then, according to the model of the streamer growth, some of them can be turn to the next state (S2). According to the model for the leader growth, some of the cells can be turn from the state S2 to the third state (S3). This procedure was repeated until the lightning approaches the ground.

At every time step only the cells, which are contiguous to the electrode surfaces ("cloud" or "ground") or to streamer and leader structures could change state (from S1 to S2). On the other hand only the cells which are contiguous to the electrode surfaces or to the leader structure could change their state from S2 to S3. Using this approach we tried to simulate the sequence of streamer – leader formation together with the step by step leader propagation of the lightning.

3. MODELS OF STREAMER FORMATION

We also use the assumption that the lightning (leader structure) can be considered as equipotential because of its very high conductivity, while the branches of streamers have very small conductivity and practically don't influence the electric field potential distribution. Thus, the absence of complete charge relaxation along the branches was modeled by neglecting of streamer conductivity. In this case the electric field potential outside the leader structure can be obtained by solving Laplace equation with boundary conditions on electrodes and leader structure.

Two assumptions were made for the growth of the streamer. Firstly the growth is stochastic in time and secondly the probability of a streamer formation is proportional to some function of local electric field $r(E)$, depending on properties of the air. This function is closely complied with the velocity of streamer tip propagation in local electric field E in front of it i.e. $u(E) = h\, r(E)$ [5].

For streamer growth criterion we used two multi-element models in which several conductive bonds can arise in each time step. One of them is the Field Fluctuation Criterion (FFC) [3-5] for the growth of a new conductive phase

$$E_i > E_* - \delta \ . \tag{1}$$

Here E_i is the local electric field in each lattice site. The parameter E_* depends on typical values of the humidity and air density. A random value δ is assumed to take into account uncertainties of atmospheric conditions, initial ionization, inhomogenities in air, thermal and other fluctuations, including fluctuations of local microfields acting on the molecules.

The probability distribution for fluctuations δ is the following

$$f(\delta) = \frac{\exp(-\delta / g)}{g} \ , \tag{2}$$

that is $\delta = -g\ln(\xi)$. Hereinafter ξ will be a random number, which is uniformly distributed in the interval from 0 to 1. In this case the function $r(E)$ has the form

$$r(E) = Ae^{E/g} \text{, where } A = \frac{1}{\tau} e^{-E_*/g} \ . \tag{3}$$

To each random value δ_i corresponds the time of appearance of the i-th bond

$$\tau_i = -\frac{\ln(1 - \exp(-\delta_i / g))}{r(E_i)} \ . \tag{4}$$

Thus, the condition (1) for a new bond to grow is equivalent to inequality $\tau_i < \tau$.

The second model we used was Multi-Element Stochastic Time Lag (MESTL) model proposed in work [4]. This new multi-element model is based on the single-element Biller's model [6] in which statistical time lags τ_i are calculated for all candidate bonds

$$\tau_i = -\ln(\xi_i) / r(E_i) \ , \tag{5}$$

and then the physical time interval is given as $\tau = \min\{\tau_i\}$. In MESTL model the physical time interval τ is chosen arbitrary and all the bonds that have $\tau_i < \tau$ arise. This model in difference of FFC model allows one to choose an arbitrary dependence of the growth probability function $r(E)$ on electric field including the form (3).

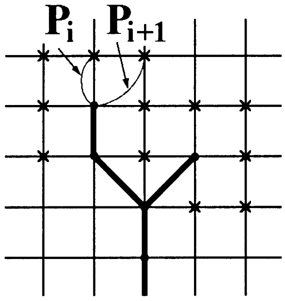

Figure 1. Possible new bonds of streamers in discrete stochastic models with the limitation of the growth TGCR.

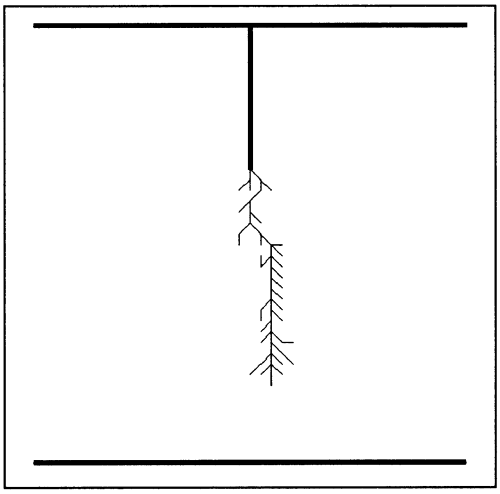

Figure 2. The MESTL model without using of TGCR. It is shown both leader and streamers branches. The initial mean electric field $E_0 = 0.3$, $t = 650$.

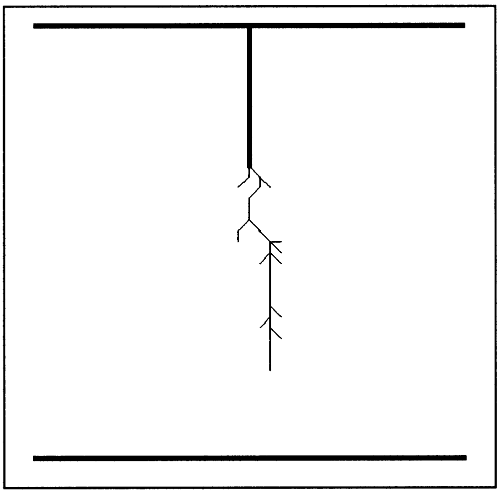

Figure 3. The MESTL model without using of TGCR. It is shown only the leader branches. $E_0 = 0.3$, $t = 650$.

The currently available high-speed photographs of the lightning phenomenon show that the lightning growth usually occurs only from the tips of existent branches. So, in some calculations a similar limitation of the growth was also used. It can be named "Tip Growth Criterion with Ramification" (TGCR) [7,8]. The arising of the new branches at each time step is allowed only from the tips of existent conductive structure (Fig. 1).

The probabilities of arising new bonds P_i depend on local electric field. The ramification of branches may have happened in this case with some probability.

4. MODEL OF LEADER FORMATION

After the formation of the streamer channels ahead of the last leader tip, there is an increase of the current of these filamentary channels as its length increases. When in some filament the current achieve sufficient value, this filament transforms into an arc, shorts circuits the other filaments and becomes a leader [9]. The physical mechanism of the streamer to leader transition is not very clear, although several theories have been proposed.

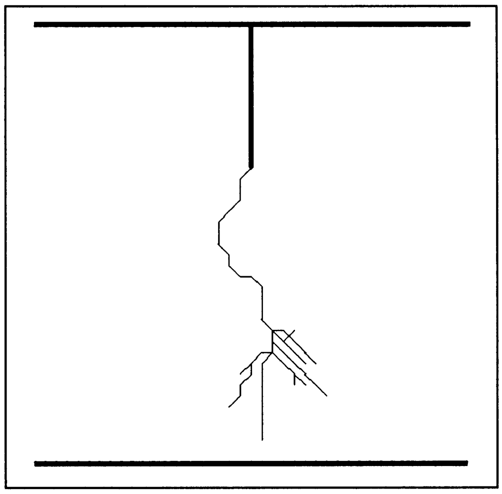

Figure 4. The MESTL model with using of TGCR. It is shown both leader and streamers branches. The initial mean electric field was $E_0 = 0.2$, $t = 500$.

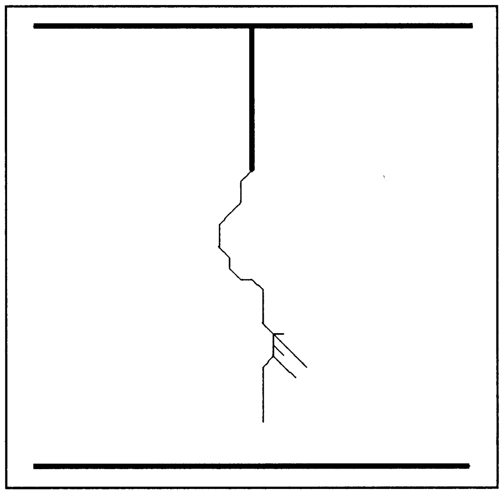

Figure 5. The MESTL model with using of TGCR. It is shown only the leader branches. $E_0 = 0.2$, $t = 500$.

In this paper we used the theory proposed by I. Gallimberti [10], in which it is of importance the energy input due to the current flow in the streamer filaments. It was considered that this energy is stored mostly as a vibrational energy in the molecules, and then the relaxation to the thermal equilibrium does occur. Relaxation time constant depends on gas temperature and absolute humidity. After some delay the increase of the temperature causes thermal detachment of ions in the streamers, with consequent increase of the stem conductivity.

Thus, streamer-leader transition occurs because of the energy of electric field transfer to molecules by electron collisions due to the current flow and then this energy during some relaxation time transforms into thermal energy. If we consider a small segment of the streamer as a cylinder with height h, cross-section S, and the conductivity σ (very small value), then the energy which is released in a time interval t will be

$$W_i = h \cdot S \cdot \sigma \int_{t_i}^{t} E(t)^2 dt \ , \tag{6}$$

where t_i is the moment of time when this bond arose.

So, if the energy released is larger than some critical value, a new leader segment is formed. This means that the criterion of the formation of a new leader segment could be

$$A \int_{t_i}^{t} E_i^2 \, dt > W_* \ , \tag{7}$$

where W_* is some critical value of energy release, and $A = h \cdot S \cdot \sigma$.

As the leader tip is approaching the ground it is expected that a return strike will be developed. The formation of this return strike is occurred due to the electric field enhancement in the gap between the tip of the leader and the ground. Of course, any protrusion on the ground increases the electric field in this gap and enhances the probability of return strike origination from its top. Anyway, the appearance (or not) of the return strike as well as the random position of its origination on the ground are stochastic and were realized in the model.

5. CALCULATIONS

The problem was solved for model geometry in that lightning occurs between two electrodes. The bottom electrode (the ground) was at the electric potential $\varphi = 0$ and the upper one (the cloud) at $\varphi = V_0$, where V_0 is considered the initial voltage difference between cloud and ground. Periodic boundary conditions in the x direction were used. The mean initial electric field in the gap was $E_0 = V_0/d$, where d is the length of the air space between cloud and ground.

The simulation was carried out in the rectangular area on lattices up to 200×200. The electric field in the region outside of the leader structure was calculated at every time step by solving the Laplace equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \tag{8}$$

with the boundary conditions on electrodes and leader structure.

At every time step new streamers may arise from the tips of existent conductive structure and the transition of one or more streamers to leader may occur. This procedure continues until the leader approaches the ground.

Eight permissible directions (including diagonals) of channel propagation was used at each site of a square lattice to diminish the anisotropy of the growing structure.

The special procedure was used to delay the growth of diagonal bonds to the next time step with the fixed probability $p = 2 - \sqrt{2} = 0.586$ [5]. At this value the mean streamer propagation velocity in diagonal direction

$$< u > = \left(\sqrt{2}h(1-p) + \frac{\sqrt{2}h}{2} p \right) r(E) = h\, r(E) \, . \tag{9}$$

Hence, streamer propagation velocity was ensured equal for all bonds including diagonals, provided that the projection of mean electric field to the corresponding direction was the same.

The reliable physical or experimental data must be used to choose the complimentary set of scales for space, time and electric field (or voltage). However, the decision has be made not to use such data but to develop a general model which may be used for simulation either of lightning discharges or of long air gaps breakdowns. These three scales can be defined in each particular case, which has to be simulated. Consequently in this study hereinafter some arbitrary units are used for space, time and electric field.

6. RESULTS

In every calculation we observed the statistical time lag of lightning origin. The mean time lag is closely related to the probability of lightning origin in time. At given geometry it sharply depends on the voltage difference.

The growing conductive structure consists of many individual streamers that propagate in a competitive way. Some of them subsequently transformed into the leader steps. A short pulse of current accompanied each leader step.

In the figures 2 and 3 it is shown the typical results of lightning simulation in the MESTL model without using of TGCR. The probability function for this model was $r(E) = (E / E_*)^n$. For these calculations the parameters were $n = 7$, $E_* = 1$. The electric field is measured in arbitrary units, as already has been mentioned above.

In the figures 4 and 5 the similar results are shown for the same model with the limitation of the growth TGCR (growth only from the tips).

In the last case one can see clearly that the growth of small-scale patterns of streamer branches does not occur from leader stem at initial stage of "lightning" propagation.

For comparison in the figure 6 the results obtained with FFC model are shown. For these calculations the parameters of FFC model were $E_* = 1$ and $g = 0.07$. The initial mean electric field was $E_0 = 0.2$.

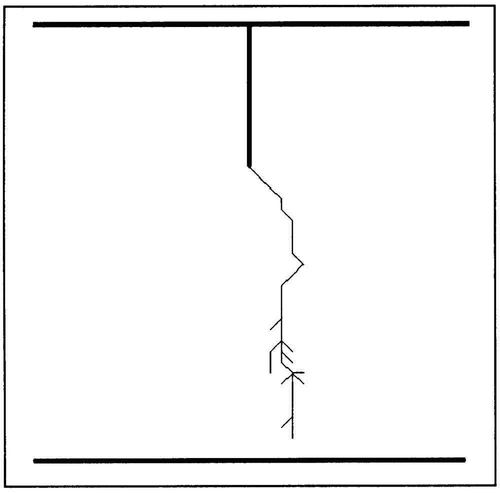

Figure 6. The FFC model with using of TGCR. It is shown both leader and streamers branches. $t = 1450$.

The initiation and growth of lightning from the cloud is shown in the figure 7. The initial mean electric field was $E_0 = 0.25$.

The formation of the return strike was observed in several simulations just before the lightning approached close to the ground.

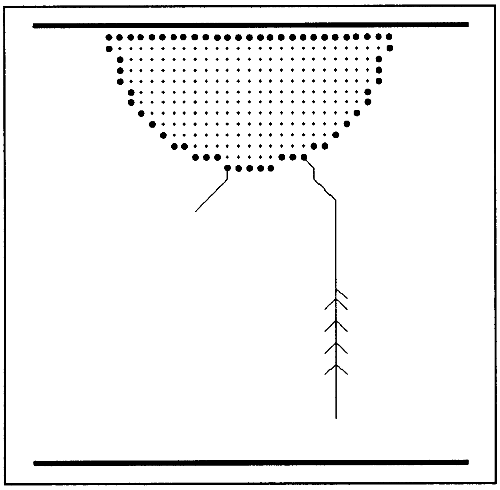

Figure 7. The FFC model with using of TGCR. It is shown only the leader branches. $t = 3360$.

7. CONCLUSIONS

The discrete stochastic model of lightning growth proposed here describes adequately main stochastic features of lightning (for example, statistical time lag and random place of lightning origin, asymmetry and non-reproducibility of detailed conductive structure, tooth-like shape of current and light pulses, return strike, etc.). This model can be useful for computer simulation of this phenomenon and very promising for application in the lightning protection technology.

The results obtained with FFC model qualitatively not differ from results obtained with MESTL model (figures 4 and 6). It confirms the equivalence of these two criteria of streamer growth.

8. ACKNOWLEDGEMENTS

This work was supported in part under a grant of the NATO Science Fellowship Program.

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