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Simulation of breakdown in air using cellular automata with streamer to leader transition

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Abstract
We propose a new discrete stochastic model for computer simulation of the lightning process and the breakdown process in long gaps in gaseous dielectrics. In this model we used cellular automata. Two different states of a conductive structure that correspond to streamers and channels of very high conductivity (leader phase) are introduced. The conductivity of the streamers is assumed to be negligible in comparison with highly conductive channels. The electric-field potential is obtained by solving the Laplace equation in a region outside the equipotential, highly conductive part of the structure. As the streamer growth criteria we use two multi-element models in which several conductive bonds can arise at each time step. The growth is stochastic in time, and the probability of streamer formation is proportional to a certain function of local electric field $r(E)$ that depends on the properties of the dielectric. If the energy released in a segment of the streamer is larger than a certain critical value, the streamer transforms to a highly conductive phase. Patterns of conductive trees are obtained in computer simulations of breakdown under various conditions.

Nomenclature

E	Electric field strength
$r(E)$	Growth probability function for streamer
p	Probability of streamer growth
n	Index in power-law approximation of growth rate function
τ	Time step in the discrete model of streamer growth
t	Current time
Δt	A time interval
$P(t)$	Time distribution of probability density for appearance of new bond of a streamer
$p(E)$	Electric-field dependence of probability of streamer growth
$\langle u \rangle$	Mean velocity of streamer tip propagation
h	The length of a bond of rectangular calculation grid (lattice)
ξ	Random number uniformly distributed in the interval from 0 to 1
E_*	A fixed value of electric-field strength. Breakdown strength of a dielectric, and also parameter of field fluctuation criterion of streamer growth
δ	Random variable having physical sense of electric-field fluctuation in a dielectric
$f(E)$	Probability density function for random variable δ
g	Characteristic width (scale) of exponential distribution for random variable δ
A	Constant at the exponential electric-field dependence of growth of streamer tip having a unit of velocity
S	Cross section of streamer segment
σ	Conductivity of streamer
W	Energy released in a streamer
W_*	A certain critical value of energy release in a streamer
B	A constant
w_*	Reduced critical value of energy release in a streamer segment

E_{th}	Threshold electric-field strength at which energy release in a streamer is insufficient for streamer-to-leader transition
$S_{i,j}$	An array of cellular automaton states
φ	Electric field potential
V_0	Applied voltage
p_-	The probability that new bond (streamer segment) does not arise at current time step
$E(x)$	Distribution of electric-field strength along the surface of grounded electrode

1. Introduction

Breakdown in solid, liquid and gaseous dielectrics is very important in designing power systems because it determines limitations of insulation in cables, transformers, electrical rotating machinery, etc widely used in electric power supply. Breakdown is a physical phenomenon with complex evolution. Thus, a description of this phenomenon is difficult because many different factors should be taken into account. Description becomes even more difficult when trying to simulate the evolution of lightning.

Up-to-date computer simulations of streamer growth are based on the idea of space and time discretization. New linear segments of the streamer channels join sequentially neighbour sites of a certain spatial lattice to the conductive structure. Thus, the shape of the conductive tree is represented by a connected graph consisting of conductive bonds (cells).

Stochastic models are widely used to simulate the propagation of a conductive phase before breakdown in solid, liquid and gaseous dielectrics. Among the first stochastic models proposed for computer simulation of breakdown in dielectrics was the Niemeyer–Pietronero–Wiesmann (NPW) model [1]. In the NPW model, the probability of streamer growth p is first related to the local electric field as $p \sim r(E)$. A conductive tree is assumed to be equipotential, and the potential distribution in dielectric is obtained from the solution of the Laplace equation outside the growing structure. Moreover, the model uses the power-law form of the growth probability function $r(E) \sim (E/E_*)^n$. This model has been employed in numerous studies for inter-electrode gaps of various geometry in two- and three-dimensional cases [2–6]. But in all these works, the authors were forced to take very small values of the index n (from 0.25 to 4). It was impossible to use sharper dependences on electric field because the formation of new segments of streamers proceeds mainly in the direction of the field and the growth of conductive structures looks like a well-directed line [6]. On the other hand, for relatively small values of n (slowly increasing function $r(E)$), the structure obtained in simulations consisted of many small branches. However, this was inconsistent with experimental observations of breakdowns in long gaps and records of lightning. The projection of electric field onto lateral direction is relatively low. Hence, for small values of n, the probability of formation of several new segments from the streamer tip becomes comparatively larger than that for sharp dependence $r(E)$.

Wiesmann and Zeller [7] attempted to solve this problem using cut-off for the growth probability function for a critical magnitude of the electric field E_{cr} that must be exceeded for the streamer to grow at a given point. This suppresses the growth of small lateral branches in the dielectric between the branches of the internal part of the conductive structure. Another approach was proposed by Femia *et al* [8], who used a special graphic post-processing for this purpose. As the measure of channel luminosity, they introduced the width of the branches and considered it to be proportional to the logarithm of the charge flow. Thus, they eliminated the narrowest branches of the patterns obtained in order to produce realistic shapes of conductive structures.

Actually, the problem is in the absence of complete charge relaxation along the branches of conductive structures. In other words, the charge relaxation is gradual rather than instantaneous, and the electric field increases slowly. This process prevents the electric field from enhancing to very high magnitudes in front of the streamer tips because the next segments of the streamer have enough time to arise at low electric field. Thus, the streamer tips propagate in a local field that is greater than the initial electric field. However, the coefficient of field amplification ahead of the streamer tip is usually in the range from 2 to 5. The last circumstance makes it possible to use an approximate approach [9] to simulate the main stochastic features of breakdown in dielectrics. In [9], a stochastic cellular automaton was used instead of exact calculations of the electric field in the absence of complete charge relaxation.

In [7, 8, 10–12], the NPW model was supplemented with the postulate of a constant voltage drop (of order of 10 kV cm^{-1}) along conductive channels to describe the stop length of a conductive structure at relatively small values of the applied voltage. In some sense, this approach also simulates the slow charge relaxation along the conductive structure, which reduces the potential of the branches and, hence, the local electrical field in the vicinity of the branch tips. In reality, this electric field must increase in time as long as the charge relaxation develops. Hence, the postulate of a constant voltage drop fails to take into account the temporal evolution of charge relaxation along the branches. This is the main disadvantage of this approach.

To take into account charge relaxation exactly, it is necessary to solve Poisson's equation together with the equation of electric charge flow along the branches of finite conductivity [13–15]. This approach includes the complicated description of state of plasma in the discharge channels taking into account the dynamics of dissociation, ionization and recombination of particles in the plasma of the channels. A full description must also include the effects of energy dissipation (expansion of channels, light emission, etc). These processes ensure the mechanism of feedback during development of a tree structure with branches of finite conductivity. In computer simulations of the growth of conductive structure, the calculation of charge transfer along the conductive branches is even more difficult [14].

The new stochastic model proposed in the present paper to simulate the formation and growth of a conductive tree does not consider all of these physical processes. Instead, for simplicity, we introduce two different states of a conductive structure: channels of very low conductivity that correspond to streamers, and channels of very high conductivity that correspond to leaders. Indeed, the structure of a leader discharge in gases

consists of conductive regions of two types: highly conductive channels (leaders) and the corona sheath (streamer zone) [16–18]. The processes inside the leader channels depend on the configuration and growth of the streamer branches. Transition of the streamer to the highly conductive phase can occur after a delay, which depends on the energy release. This allows us to describe the rise in potential at the tips of the conductive branches with time and, hence, to take into account charge relaxation along the branches in a first approximation. Our two-stage model is simple enough compared to that of Dul'zon *et al* [15], who assumes that the conductivity of the channels is proportional to the internal energy of plasma.

In [14] it was proposed that all possible stochastic models of streamer growth be divided into two groups. From a physical point of view, models of the first group assume that the growth of the first bond (streamer) suppresses the development of the others at a current time step. Therefore, just one bond is added in a time step (single-element models). In the second group of models, in contrast, the appearance of any bond does not influence the development of the others. Thus, in models of the second group, several bonds can be generated at each time step (multi-element models). For models of the first group, the time step is equal to the delay required for the appearance of a first new bond. The sequence of time intervals calculated in a proper way for each growth step according to certain rules is called 'physical time'. For all models of the second group, the time step is constant and all bonds that may arise in a shorter time than the time step are accepted.

Let us consider the generation of a new bond at a current time step of duration τ as a stochastic process in time. The time step τ of the multi-element model can be divided into smaller intervals of duration $\Delta t \ll \tau$. It is assumed that (1) the random event of this bond generation does not depend on the generation of other candidate bonds, provided that the local electric field E is constant (2) the probability of this bond generation at time t is independent of the events that precede t and (3) the probability of bond generation in a short time interval Δt is $p = r(E)\Delta t$.

The probability of a random event of bond generation at time t is obtained from Bernoulli's statistical test scheme. Using Poisson's theorem of the theory of stochastic processes, we can obtain the following probability density for the appearance of a bond [19]:

$$P(t)\,\mathrm{d}t = r(E)\exp(-r(E)t). \tag{1}$$

For all multi-element models in every time step τ, the streamer may travel distance h with probability

$$p(E) = 1 - \exp(-r(E)\tau) \tag{2}$$

and may stand still with probability $1 - p(E)$. Here h is the bond length in the calculation grid. Hence, the mean propagation velocity is given by

$$\langle u\rangle = hp(E)/\tau \tag{3}$$

provided that the electric field ahead of the streamer tip is equal to E. In a low electric field, the probability of streamer growth (2) is small and $p(E) \approx r(E)\tau$. It follows from (3) that $\langle u\rangle = hr(E)$ [13] in this case. Thus, the function $r(E)$ is closely related to the average velocity of streamer tip propagation in the local electric field E ahead of the tip.

In 1993 Biller [20] proposed the first single-element model, in which 'physical time' was calculated correctly. He assumed that the probability density (1) is valid for the delay time of generation of all candidate bonds t_i, and hence all possible t_i were calculated from the formula

$$t_i = -\ln(\xi_i)/r(E_i). \tag{4}$$

Hereafter, ξ will be a random number that is uniformly distributed in the interval from 0 to 1. Just one bond that has the shortest delay time was added to the streamer structure. Precisely this quantity $\tau = \min\{t_i\}$ was assumed as the physical time interval.

To simulate the breakdown process, we used cellular automata (CA), a well known method for simulating the evolution of complex natural systems [21–23]. Different CA have been applied to physical and technological problems involving local interaction. Despite their structural simplicity, CA can exhibit a complex dynamic behaviour and can describe many physical systems and processes.

Danikas *et al* [24, 25] used CA for simulation of streamer structure growth. They employed the simplest deterministic criterion of growth $E_i > E_*$ (field threshold criterion (FTC)), where E is the local electric field and E_* is the breakdown strength of the dielectric. Stochastic features were introduced into the model by means of random values of the dielectric constant ε, uniformly distributed in the range between 2.1 and 2.25. Unfortunately, this approach does not meet the requirements for streamer growth criteria with physical time that were analysed in detail in [14].

2. Models of streamer formation

We assume that part of the conductive structure is equipotential because of its very high conductivity. On the other hand, the streamer branches have very small conductivity and practically do not influence distribution of the electric field potential. Thus, the absence of complete charge relaxation along the branches was modelled by neglecting the streamer conductivity. In this case, the electric field potential outside the highly conductive structure can be obtained by solving the Laplace equation with boundary conditions on the electrodes and the highly conductive structure.

Two assumptions were made for streamer growth. First, the growth is stochastic in time and, second, the probability of streamer formation is proportional to a certain function of the local electric field (2) that depends on the properties of the dielectric.

In the present work as the streamer growth criteria, we used two multi-element models in which several conductive bonds can arise in each time step. The first is the field fluctuation criterion (FFC) [9, 13, 14] for the growth of a new conductive phase. If the condition

$$E_i > E_* - \delta \tag{5}$$

was fulfilled in a cell, then the new segment of the streamer forms there during time step τ. Here E_i is the mean local electric field in each lattice cell. The parameter E_* depends

on the properties of a particular dielectric. The random variable δ (fluctuation) is assumed to take into account the uncertainty in the value of E_* due to inhomogeneities in the dielectric and thermal and other fluctuations, including uncertainties in the external conditions (for example, initial ionization of the dielectric). In the case of modelling lightning processes, one should also take into account many different and, sometimes, unknown factors (such as air density and humidity, atmospheric ionization, etc). However, all these uncertainties of the lightning phenomenon can be treated as stochastic features already included in the model. In this sense, the new stochastic model is also suitable for simulation of the initiation and growth of the lightning tree.

Let us consider one candidate bond on which a new segment of the streamer may arise at a current time step τ. For electric field $E < E_*$, the probability that this streamer segment appears at time $t < \tau$ is equivalent to the probability of satisfaction of inequality (5):

$$p(E) = \int_{E_*-E}^{\infty} f(\delta)\,\mathrm{d}\delta. \tag{6}$$

Here $f(\delta)$ is the probability density function for the random variable δ. The particular form of the function $f(\delta)$ can be determined from (3) using experimental data on the electric-field dependence of the mean propagation velocity of the streamer.

The values of probabilities (2) and (6) can be equated to determine the value of the time step τ. Thus, for all models involving the growth criterion (5), the value of τ can be expressed in terms of the functions $r(E)$ and $f(\delta)$:

$$\tau = -\frac{1}{r(E)} \ln\left(1 - \int_{E_*-E}^{\infty} f(\delta)\,\mathrm{d}\delta\right). \tag{7}$$

Suppose that each random value of δ corresponds to a definite time t when the new bond should arise. In this case, all values of δ that satisfy inequality $\delta > E_* - E$ correspond to the condition that a new bond should arise at time $t < \tau$. On the other hand, all values of δ that satisfy the inequality $\delta < E_* - E$ correspond to the condition that a new bond should arise at time $t > \tau$. It is obvious that value of $\delta = E_* - E$ corresponds to $t = \tau$.

Hence, from (7) we obtain the following relation between the random moment when the new segment arises t and the random variable δ:

$$t = -\frac{1}{r(E)} \ln\left(1 - \int_{\delta}^{\infty} f(\delta)\,\mathrm{d}\delta\right). \tag{8}$$

In this paper, we use an exponential probability density for fluctuations δ as a particular case:

$$f(\delta) = \frac{\exp(-\delta/g)}{g} \tag{9}$$

which is equivalent to $\delta = -g\ln(\xi)$. The physical meaning of g is the characteristic width (scale) of the distribution (9) of the random variable δ. For this exponential distribution, the value of g is also equal to the mean value of the random variable δ. It follows from (3) and (6) that the mean velocity of streamer propagation $\langle u(E)\rangle$ in electric field E is given by

$$\langle u(E)\rangle = A\exp(E/g) \qquad \text{where} \quad A = \frac{h}{\tau}\exp(-E_*/g). \tag{10}$$

Thus, for the FFC model, the definite time of appearance of the ith bond corresponding to each random value δ_i [14] is equal to

$$t_i = -\frac{\ln(1 - \exp(-\delta_i/g))}{r(E_i)} \tag{11}$$

where $r(E)$ is obtained from (7) and (9):

$$r(E) = -\frac{1}{\tau}\ln(1 - \exp(-(E_* - E)/g)). \tag{12}$$

Thus, the field fluctuation criterion of streamer growth (5) is equivalent to the growth criterion according to which all bonds that have $t_i < \tau$ should arise.

The second model we used was the multi-element stochastic time lag (MESTL) model proposed in [14]. This new multi-element model is based on Biller's single-element model [20]. In the MESTL model, the physical time step τ is arbitrary. All possible bonds should arise that have delay time of appearance t_i, calculated from (4), less than τ. The MESTL model, in contrast to the FFC model, allows one to choose an arbitrary dependence of the growth probability function $r(E)$ on the electric field including form (12).

From (9) and (11) it follows that random values of the delay time for our FFC model can be calculated from the formula $t_i = -\ln(1-\xi_i)/r(E_i)$. This formula is equivalent to formula (4) provided that the random numbers ξ_i are uniformly distributed in the interval from 0 to 1. Hence, the FFC and MESTL criteria of growth are shown theoretically to be equivalent (see also [14]).

3. Model of streamer transformation to a highly conductive phase

When, in a certain filament, the released energy reaches a sufficient value, this filament transforms into a highly conductive arc. The physical mechanism of streamer transition to a highly conductive phase is not very clear, although several theories have been proposed [26, 27].

Gallimberti [26] developed a theory in which the energy input due to the current flow in the streamer filaments is of importance. He assumed that the energy of electric field is transferred to molecules by electron collisions and is stored mainly as vibrational energy in the molecules. This means that in a new segment of the conductive channel the plasma is initially in a non-equilibrium state because the vibrational temperature is much higher than the translational temperature [27]. Then relaxation to thermal equilibrium occurs simultaneously with heating of the plasma. The relaxation time constant depends on the gas temperature. After a short delay time at the final stage of transition of the streamer to a spark (highly conductive phase), the process of V–T relaxation accelerates because the plasma temperature increases, leading to a subsequent increase in the stem conductivity. Thus, transition of the streamer to a highly conductive phase occurs. The transition time depends sharply on the voltage drop along the channel. For example, for dry air, the transition time varies from 10^3 to 10^2 ns as the corresponding electric field increases from 20 to 24 kV cm^{-1} [27].

In this paper, we use a somewhat simplified approach. If we consider a small segment of the streamer as a cylinder with

height h, cross section S and conductivity σ (very small value), then the total energy released by time t is

$$W_i = h \cdot S \cdot \sigma \int_{t_i}^{t} E(t)^2 \, \mathrm{d}t \tag{13}$$

where t_i is the time when this bond arose.

Thus, if the released energy is greater than a certain critical value, a new highly conductive segment is formed. This means that a criterion for the formation of a new highly conductive segment can be

$$B \int_{t_i}^{t} E_i^2 \, \mathrm{d}t > W_* \tag{14}$$

where W_* is a certain critical value of the released energy and $B = h \cdot S \cdot \sigma$. It is convenient to use the parameter $w_* = W_*/B$. Thus, the segments of highly conductive channel are formed after a certain time delay that depends on the internal electric field and, hence, on the applied voltage. In the limit $w_* = 0$, our model has only a highly conductive phase and behaves like the ordinary Laplace model. It should be noted that we do not consider how 'reverse' processes such as light emission, hydrodynamic expansion, etc influence the streamer dynamics. Nevertheless, we introduced some restriction on the condition of the streamer-to-leader transition. In addition to (14), we assume that in the channel the current local electric field must exceed a certain minimal field E_{tr} in order to ensure sufficient energy release and, hence, to prevent the decay of plasma by these 'reverse' processes.

4. Cellular automata

In this paper, we extend the NPW approach to simulation of breakdown [1] and propose a new stochastic model that takes into account the growth of the initial streamers and subsequent transitions of some of them to highly conductive channels. For this purpose, we use CA.

The automata are a class of very powerful models originated in computer science. They are well known extensions of the 'classical' automata. They provide mathematical models for a wide variety of complex natural phenomena, from growth of patterns in biological systems to turbulence in fluid dynamics. A cellular automaton consists of

- a lattice of cells; this lattice can be two- or three-dimensional and have arbitrary size. At each time, each cell can be in one of a finite number of distinct states.
- local rules of transformation from one state to another, which depend only on the states of the neighbour cells.

The states of all cells are updated simultaneously at each time step according to the so-called local rules of CA.

In the present work, a two-dimensional stochastic CA was employed, and the space between the electrodes was divided into cells. Thus, an array of states of the cellular automaton $S_{i,j}$ was used. As in [24, 25], we employed the so-called extended form of CA, in which not only the discrete states of CA but also some real physical values (electric potential, energy release, random fluctuations, etc) were kept in each cell. Hence, our stochastic CA transformation rules define a new state of a cell at a next time step using not only data on the states of the CA but also information on the physical parameters in a local neighbourhood of the cell considered:

$$S_{i,j} = \Phi(S^*, \varphi^*, \delta, W_{i,j}). \tag{15}$$

Here S^* is the set of states of the CA in this and neighbour cells, φ^* is the set of values of the electrical-field potential, δ is the random value generated independently for this cell at each time step and $W_{i,j}$ is the energy released in this cell.

For development of the model, we proposed that the gradual change of the electrical conductivity along the branches be approximated by a sequence of a lowly conductive phase (streamer) and a highly conductive phase (leader). In view of this, each cell occupied by the dielectric can be in three main states. The first is the initial state. This means that the material in the cell is still a dielectric. The second state corresponds to the formation of a streamer in this cell. The third state indicates that the transition of the 'streamer' state to the 'leader' state has occurred.

Before initiation of breakdown, all cells of the dielectric were in the initial state (S0). Then, according to one of the stochastic models for streamer growth, some of the cells can turn into the next state (S1) that corresponds to the streamer. According to the model of streamer transformation to a highly conductive phase, some of the cells can transform from state S1 to states S2 or S3 that correspond to the cells of downward and upward leaders, respectively. It was assumed that a downward leader is equipotential and has the potential of the upper electrode. The same was assumed for an upward leader, which has the potential of the lower electrode.

We used the distinct states of the cellular automaton not only to describe cells that initially were in the dielectric state but also to indicate cells that belong to the upper and lower electrodes (the states S4 and S5, respectively).

An iterative process is a method for solving the Laplace equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0. \tag{16}$$

We used a procedure in which all values of the cells belonging to the dielectric (S0) and streamers (S1) must be updated in accordance with the formula

$$\varphi_{i,j} = \frac{\varphi_{i,j+1} + \varphi_{i,j-1} + \varphi_{i+1,j} + \varphi_{i-1,j}}{4}. \tag{17}$$

At every iteration step, the potentials of the cells of the lower electrode (S5) and the upward leader (S3) must be set equal to zero. Moreover, we must assign the value φ_0 to the potentials of the cells belonging to the upper electrode (S4) and the downward leader (S2). Such formalization simplifies significantly the logical structure of the computer program. It allows us to consider all cells in a standard way. For example, in PASCAL language, it is possible to use the following simple operator during solution of the Laplace equation:

```
case S[i,j] of
   0,1: F[i,j]:= (F[i+1,j] + F[i-1,j]
       + F[i,j+1] + F[i,j-1])/4;
   2,4: F[i,j]:=F0;
   3,5: F[i,j]:=0;
end;
```

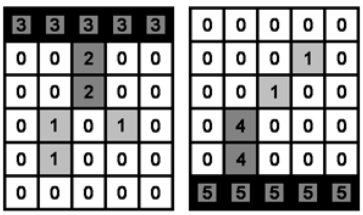

Figure 1. Examples of conducting tree growth from upper and lower electrodes. The numbers in the cells are the states of our CA.

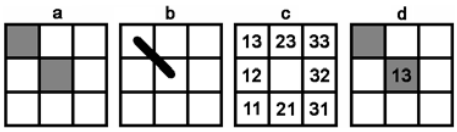

Figure 2. (a), (b) Example of the nearest neighbourhood of one of the tips of the downward streamer. The grey cell at the centre originated from the cell in the left upper corner. (c) Local coordinate system in the vicinity of the central cell. (d) The information stored in this cell indicates the direction from which a new conductive bond originated.

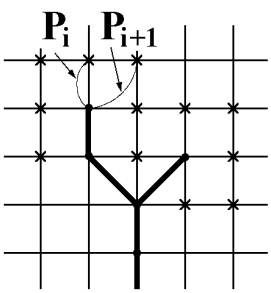

Figure 3. Possible new bonds of streamers in discrete stochastic models with the GOT limitation.

where the letter F represents the electric-field potential φ. The configuration of the electrodes was defined initially by assigning values '3' or '5' to some elements $S_{i,j}$ of our array of cellular automaton states (figure 1).

At every time step, a change of state from S0 to S1 is possible only for cells contiguous to the electrode surfaces or the streamer or the highly conductive phase. On the other hand, only cells contiguous to the electrode surfaces or the highly conductive phase could change their state from S1 to S2 or S3. The procedure proposed is repeated until the conductive tree approaches the opposite electrode.

Of course, in reality, the conductive tree consists of small linear segments (figure 2(b)) rather than of square elements (figure 2(a)). To describe this, we introduced a local coordinate system in the vicinity of each cell (figure 2(c)). The first digit of the number in each neighbour cell denotes the x coordinate of the cell (column number), and the second digit denotes the y coordinate of it (row number). In this case, each neighbour cell has a unique value. We place one of these values into the cell considered to indicate one of the neighbour cells from which the conductive bond originated. This information was kept in the cell as an additional set of states of the cellular automaton. For example, number '13' placed in the central cell (figure 2(d)) becomes a certain pointer which indicates that this cell (bond) originated from the cell located in the upper left corner of the nearest neighbourhood. In this case, it is possible to use this information, for example, to draw segments of the conductive structure (figure 2(b)), and to consider the structure as a graph consisting of conductive bonds (figure 3).

Should new streamer branches arise only from the leader tips or from any part of the leader? At present, this question remains to be solved. For example, during lightning propagation, new streamer branches arise mainly from the leader's tip. The same situation is observed in the breakdown in dielectric liquids. The currently available high-speed photographs of the breakdown phenomenon show that the growth of a conductive tree usually occurs only from the tips of the existing branches [28]. Therefore, a similar limitation of the growth was also used in some calculations. It was assumed that at each time step, the appearance of new streamer branches is allowed only from the tips of the existing conductive structure (figure 3) [29, 30]. The growth of new streamer segments that may originate not from the tips of the conductive structure was forbidden. This limitation of conductive tree growth can be called 'growth only from the tips' (GOT). The probabilities that new bonds arise P_i depend on the local electric field according to the growth probability function $r(E)$. In this case, ramification of the branches may occur with some probability.

5. Calculations

The problem was solved for the geometry in which breakdown occurs between two electrodes. The lower electrode was at an electric potential $\varphi = 0$ and the upper one was at $\varphi = V_0$, where V_0 is the applied voltage. Periodic boundary conditions in the x direction were used. The experiments on breakdown in long gaps were modelled in point–plane geometry with gap length d. In simulations of lightning, the 'cloud' was modelled by an equipotential surface at constant potential $\varphi = V_0$. This approach is usual for such kind of simulations.

The simulation was carried out in a rectangular area on lattices up to 600×600. In the region outside the highly conductive structure, the electric field was calculated at every time step by solving the Laplace equation with the boundary conditions on the electrodes and the highly conductive structure. To calculate the potential distribution at the next time step, we used the potential distribution at the previous time step as the initial data for the iterative process because these values did not differ significantly from one another.

At every time step, new streamers may arise from the branches of the existent conductive structure and transition of one or more segments of the streamers to the highly conductive phase may occur. This procedure was repeated until the conductive tree approaches the opposite electrode.

Eight permissible directions (including diagonals) of streamer propagation are allowed at each site of the square lattice to diminish the anisotropy of the growing structure. By analogy with formula (3), the mean velocity of streamer tip propagation in diagonal directions is

$$\langle u \rangle = \sqrt{2} hp(E)/\tau \tag{18}$$

which is greater by a factor of $\sqrt{2}$ than the mean velocity (3) for the basic directions of the grid, provided that the longitudinal

projection of the electric field has the same value. It is important to ensure the same velocity in diagonal directions provided that new bonds arise with the same probability. A possible way to do this is to delay the growth of diagonal bonds until the next time step with certain fixed probability p_- [13]. In this case, a new bond of length $\sqrt{2}h$ may arise with the same probability $p(E)$ as for non-diagonal direction, but it may arise in the current time step τ with probability $(1 - p_-)$ or may arise at the next time step (time interval is equal to 2τ) with probability p_-. Hence, the mean velocity of streamer propagation in diagonal directions has the form

$$\langle u \rangle = \left(\frac{\sqrt{2}h}{\tau}(1 - p_-) + \frac{\sqrt{2}h}{2\tau} p_- \right) p(E). \qquad (19)$$

This expression for the mean velocity of streamer propagation in diagonal directions coincides with formula (3) for the usual non-diagonal directions at value $p_- = 2 - \sqrt{2} = 0.586$. Thus, identical streamer propagation velocities were ensured for all bonds, including diagonals, provided that the longitudinal projection of the mean electric field onto the corresponding direction was the same. For this purpose, the additional state of the CA (S6) was specially used to indicate the cells of the dielectric for which the growth of diagonal bonds was delayed.

6. Results

In all simulations, hereafter we use arbitrary units. To choose a reciprocally complementary set of scales for space, time and electric field (or voltage), one needs to have reliable physical or experimental data. These three scales can be defined in each particular case that is to be simulated. This is difficult for the case of lightning but is easier for the case of breakdown in long air gaps, for which many experimental data obtained under well-defined conditions exist. However, instead of using experimental data, we decided to develop a general model that could be applicable for simulation of both the lightning phenomenon and breakdown in gaseous dielectrics. The time, length, charge, voltage and electric field are given in arbitrary units in the present work to emphasize the common features of the phenomena.

One of the most important components of every stochastic model of electrical breakdown is the growth probability function $r(E)$. It carries information on the physical mechanisms of breakdown initiation and streamer propagation. Dependence (12) of the FFC model for parameter values $E_* = 1$, $g = 0.1$ and $\tau = 1$ is shown in figure 4 (curve 1). From formula (12) it follows that in the range of electric fields up to $E = 0.8$, $r(E) \approx A \exp(E/g)$, where $A = \exp(-E_*/g)/\tau$. As noted above, the electric field is given in arbitrary units. This dependence is shown in figure 4 (curve 2). It can be considered as an alternative approximation of the possible ionization mechanisms of breakdown parallel with the simple power-law dependence $r(E) \sim (E/E_*)^n$. For some parameter values $n = 9$, $E_* = 1$ and $\tau = 1$ (figure 4, curve 5) the power-law dependence is very close to the exponential dependence (curve 2).

In every calculation, we observed a statistical time lag for the initiation of conducting trees. For fixed geometry, it depends sharply on the voltage difference. In the case of lightning, the mean time lag is closely related to the probability of appearance of lightning in time.

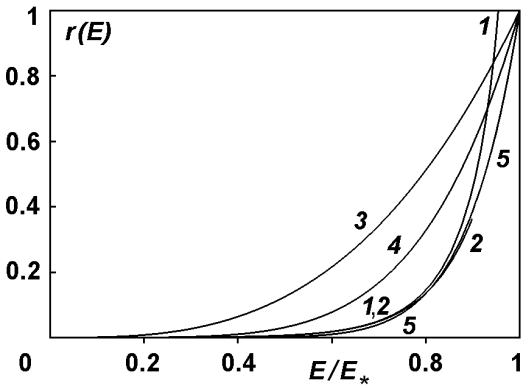

Figure 4. Streamer growth probability function $r(E)$. Curve 1 corresponds to formula (12) at $E_* = 1$, $g = 0.1$ and $\tau = 1$. Curve 2 is the exponential approximation of $r(E)$ obtained from (12), which is valid for small growth probabilities. Curves 3, 4 and 5 correspond to the power-law form ($n = 3$, 5 and 9 respectively, $E_* = 1$ and $\tau = 1$).

The growing conductive structure consisted of many individual streamers that propagated in a competitive way. Some of them subsequently transformed into a highly conductive phase. A short current pulse accompanied each of these events because of the stepwise change of the total charge of the conducting structure. For every node occupied by leaders or electrodes, one can consider Poisson's equation and determine the charge of this node using values of the potential in neighbour cells. The total charge, for example, the total charge of the downward leader and the upper electrode, was calculated by summing up the charges of the nodes belonging to them (in the present model, we neglected the conductivity of the streamers and, hence, the streamer cells cannot carry charge). The accuracy of calculation of the charge (several per cent) was checked by a comparison of the electric charges of the upper and lower electrodes, including the charge of the downward and upward leader structures, respectively. Plots of the total charge of the downward conducting structure calculated together with the charge of the upper electrode are shown in figure 5. The MESTL model with parameters $n = 9$, $E_* = 1$ and $\tau = 1$ was used. Immediately after voltage supply, every electrode has a certain initial charge proportional to the applied voltage because the capacitance of the electrode system was constant. For relatively high values of the applied voltage (curve 3), the total charge of the downward leader and the upper electrode increases rapidly because of the fast growth of the conducting structure.

Figures 6 and 7(a) show results of simulations of breakdown in point–plane geometry obtained with the FFC model. For these calculations, the parameters of the FFC model were $E_* = 1$, $g = 0.1$ and $\tau = 1$. In the last case, the value of the electric field was somewhat smaller, and one can clearly see that the growth of small-scale patterns of streamer branches does not start from the highly conductive stem at the initial stage of conducting tree propagation.

As mentioned above, the FFC and the MESTL criteria of growth were shown theoretically to be equivalent. In the present work, we compared the results obtained with

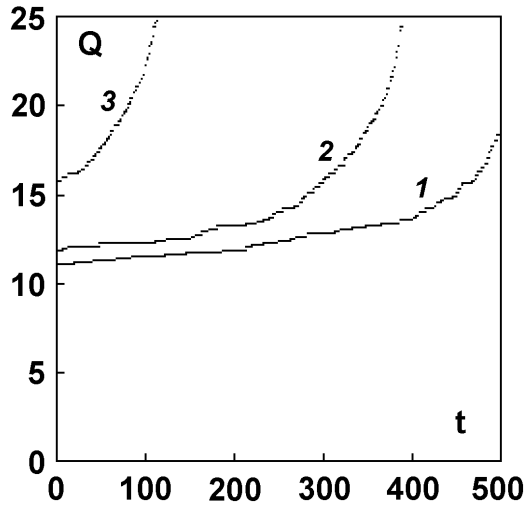

Figure 5. Total electric charge of the downward conducting structure calculated together with the charge of the upper electrode (MESTL model with $n = 9$, $E_* = 1$ and $\tau = 1$). Lattice size was 100×100. The applied voltage was $V_0 = 9.3$, 10 and 13.3 (curves 1, 2 and 3 respectively).

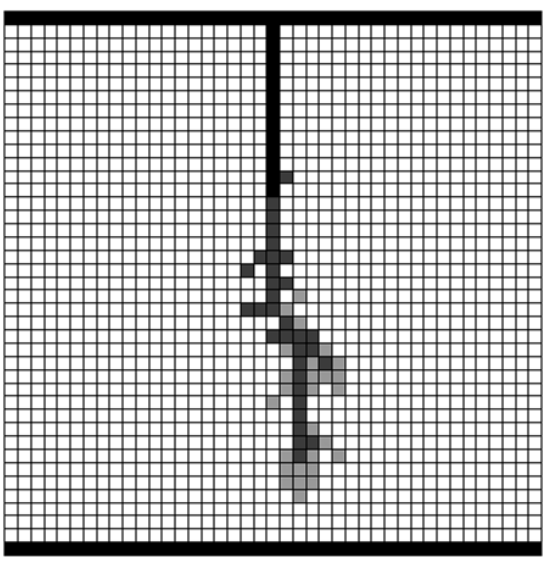

Figure 6. Typical configuration of conductive cells obtained in the FFC model with the GOT limitation. The light grey cells indicate the streamers. The dark grey cells indicate the high conductive phase. The applied voltage was $V_0 = 8.1$ and $w_* = 50$. Lattice size was 40×40; $t = 1120$.

these models and concluded that they actually produced stochastically similar patterns of conducting structures. Figure 7 shows typical results obtained for the FFC model (a) and for the MESTL model (b). For both growth criteria, the dependence $r(E)$ was used in form (12) with $E_* = 1$, $g = 0.1$ and $\tau = 1$. Moreover, the GOT limitation on the growth (growth only from the tips) was used.

The distribution of electric field $E(x)$ along the lower electrode, as downward leader approaches to it, is shown in the figure 7(b). Evidently, the maximum electric field on the surface is approximately opposite to the advanced tips of the leader structure. This effect can lead to the generation of an opposite upward leader just before the leader channel

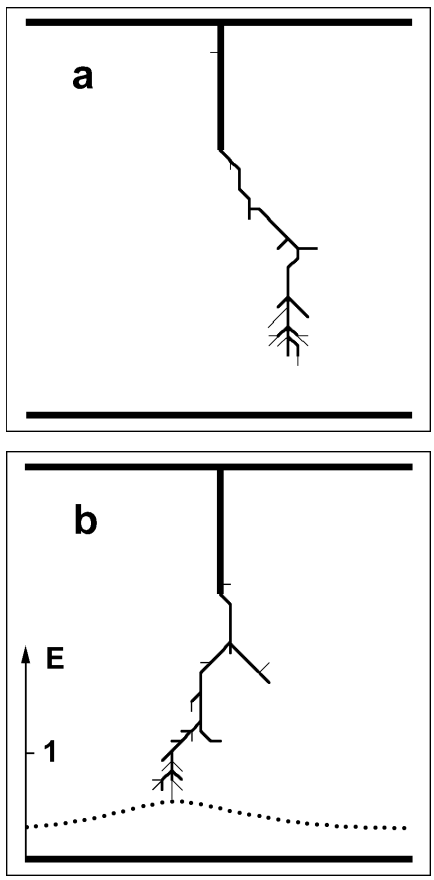

Figure 7. Typical results obtained (a) for the FFC model and (b) for the MESTL model. Here both the streamers (thin lines) and the branches of high conductivity (thick lines) are shown ($w_* = 1.1$). The electric-field distribution $E(x)$ along the lower electrode is also shown (b). The applied voltage was $V_0 = 5.4$. Lattice size was 40×40; $t = 710$ (a) and 620 (b).

approached closely the ground as was observed in several simulations. In any case, the propagation of the tips accelerates as the conductive structure approaches the ground.

The results of simulations of prebreakdown phenomena using the MESTL model are shown in figures 8–10. The model with the streamer-to-leader transition (SLT) was compared with the ordinary Laplace model (OLM) with a single, highly conductive, phase. The conditions of occurrence of lightning are governed by the quantity and density of the charge accumulated in the cloud and by the distance between the cloud and the ground. The characteristic time of change of these parameters is large enough (of the order of several tens of minutes). Hence, as a rule, breakdowns occur under low overvoltage conditions. At low applied voltage, the conducting structure in the SLT model almost entirely consists of a highly conductive phase. Hence, the generation of highly conductive structures does not differ from the results obtained using the OLM (figure 8). However, the time of structure growth is significantly greater in the SLT model because of the delay in the SL transition (figure 8), which depends on the value of w_*.

Under moderate overvoltage conditions, the development of conducting structures proceeded when the relative values of the electric field ahead of the tips E/E_* was about 0.5

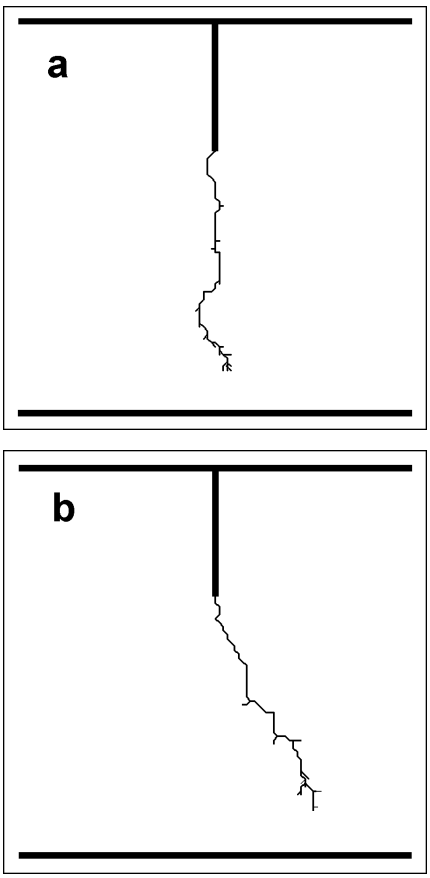

Figure 8. Results obtained for the MESTL model in the limit of a single, highly conductive, phase (a) and with SL transition at $w_* = 1.1$ (b). Both the streamers and the branches of high conductivity are shown. $n = 9$, $V_0 = 6$ and $t = 1860$ (a) and 3040 (b). Lattice size was 100×100.

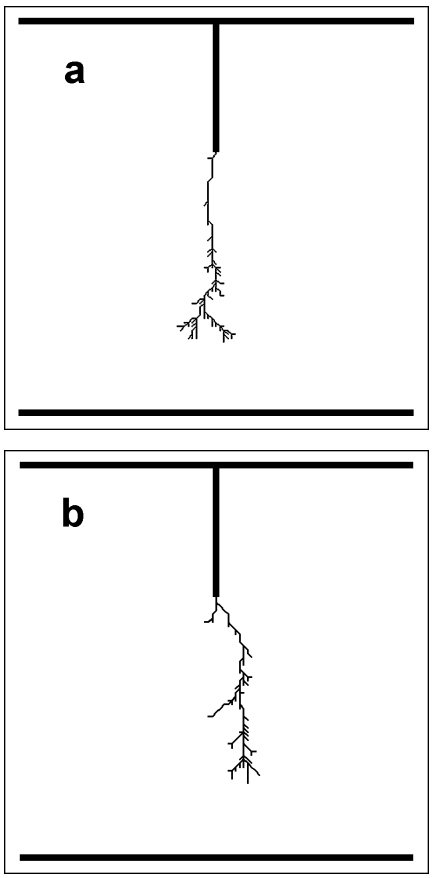

Figure 9. Results obtained for the MESTL model in the limit of a single, highly conductive, phase (a) and with SL transition at $w_* = 1.1$ (b). Only the branches of high conductivity (leaders) are shown. $n = 9$, $V_0 = 10$ and $t = 63$ (a) and 183 (b). Lattice size was 100×100.

at the beginning of growth and 0.8 at the growth stages shown in figure 9. Hence, in both cases, the conducting structures practically did not branch initially. In the case of the OLM, the conducting structure became more branching as its tips approached the ground. This effect was not so significant in the case of the SLT model (figure 9(b)) because the streamer tips propagate in the vicinity of the leaders in the lower electric field. This is a main advantage of the model proposed. One can see that in the one-phase model (Laplace growth), the velocity of the leader is approximately two times as high as in the case of the model with the SL transition.

It was mentioned above that the rate of growth is governed by the local electric field ahead of the streamer tips and depends on the growth probability function $r(E)$. To compare our results with the results obtained for the commonly used low values of the index n in the function $r(E)$, we carried out simulations with a relatively low value of $n = 3$ using the MESTL model in the limit of a single, highly conductive, phase (figure 10(a)) as well as with the SL transition (figures 10(b) and (c)). It can be seen that the patterns in both cases are more branched, as expected, than for $n = 9$ (figures 8 and 9). At the beginning of this work, we assumed that in reality only the leader structures are usually observed as bright luminous objects because energy is released mainly inside them. Hence, we considered only the leader patterns (figures 9(b) and 10(c)). The system of streamer branches carries only auxiliary information and is known as the corona sheath (streamer zone) [16–18], which is actually observed in experiments using special sensitive methods.

In the case of the OLM, as mentioned above, the leader structure (figure 10(a)) has many small branches, which grow rather chaotically not only toward the ground but often also in the backward direction. The structure obtained using the SLT model is free of this disadvantage (figure 10(c)).

Results of simulation of lightning initiation and growth from a cloud are shown in figure 11. The electric potential of the cloud was $V_0 = 13.3$. We note that the formation of the opposite upward leader was observed in several simulations just before the lightning approached the ground. Sometimes, the start of growth of the second and subsequent structures was observed at the electrode surface (figure 11).

7. Discussion and conclusions

The discrete stochastic model of conducting tree growth proposed here describes adequately the main stochastic features of breakdown (for example, statistical time lag and random place of origin, asymmetry and non-reproducibility

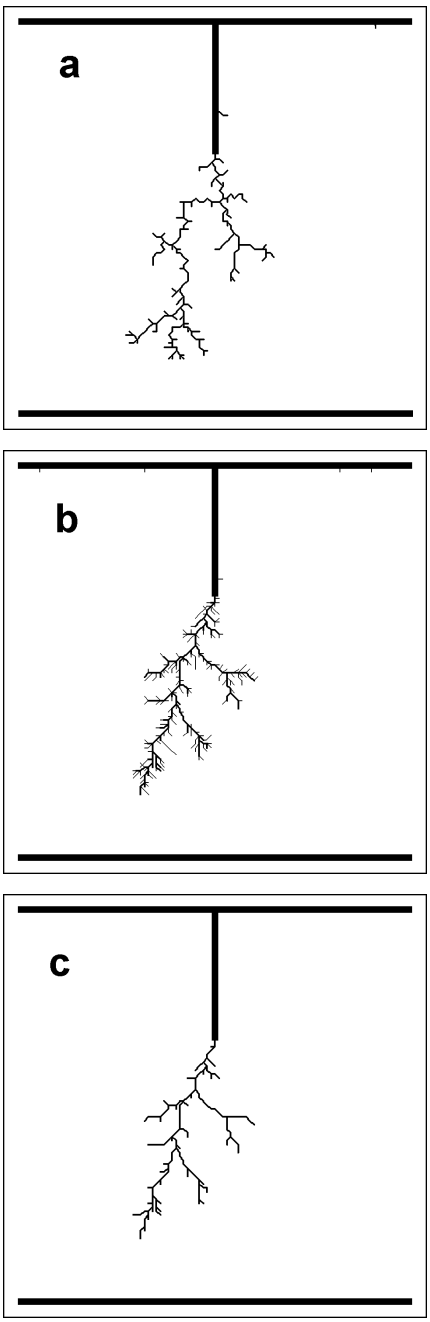

Figure 10. Results obtained for the MESTL model in the limit of a single, highly conductive, phase (a) and with SL transition at $w_* = 1.1$ (b), (c). $n = 3$, $V_0 = 6$ and $t = 180$ (a) and 330 (b), (c). Both streamers and leaders (b) and only leaders (c) are shown. Lattice size was 100×100.

of a detailed conducting structure, tooth-like shape of current and light pulses, opposite leader, etc). Thus, the results of simulations of the growth of the streamer and leader structures are in qualitative agreement with experiments.

In the two-stage model, the streamer branches have very small conductivity and thus, as mentioned above, do not influence the electric-field distribution. At first glance, it might seem that streamers could not propagate for more than one step from the leader or electrode. At the same time, we can see that streamers can pass ahead the leaders up to

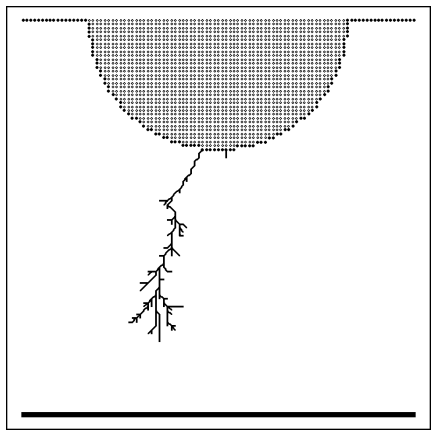

Figure 11. Typical pattern of lightning structure (the MESTL model using the GOT limitation). Only the branches of high conductivity (leader) are shown. Lattice size was 100×100; $n = 9$, $V_0 = 13.3$ and $t = 4090$.

three or four steps in our calculations (figures 6, 7(a) and 7(b) and 10(b)). Nevertheless, in the two-stage model, streamers actually cannot go far away from the highly conductive structure because there is no charge relaxation along the streamer branches and the electric field at the tip of long streamer branch is relatively low.

The results obtained using the FFC model do not differ qualitatively from the results obtained for the MESTL model (figures 7(a) and (b), respectively). This confirms the equivalence of these two criteria of streamer growth, which was theoretically proved above (see also [14]). The GOT limitation is optional and might be included in the model or not. We believe that models with the GOT limitation give more realistic patterns of conducting structures (figure 11), while the regular growth criteria lead to bushy (too dense) structures.

The model proposed here with the transition of a streamer to a highly conductive phase allows one to use a sharper dependence of the growth probability function $r(E)$ on the electric field (for example, elevated values of the index n in the power-law approximation up to 9 or formula (12)) without any cut-off procedures. A sharper dependence of the growth probability function reduces the branching of the growing conducting structure.

The model of breakdown with subsequent transition of streamers to highly conductive phase (leaders) allows one to simulate in the first approximation the effect of charge relaxation in contrast to the ordinary Laplace models, in which relaxation was assumed to be complete [1–6]. The growth rate of the total structure depends not only on the velocity of streamer propagation but also on the rate of SL transition. In some sense, the last process simulates in the simplest manner the evolution of the local electric field ahead of the streamer tips, which increases owing to gradual charge relaxation. The delay time depends on the value of w_*. For relatively low w_*, the delay time of streamer transition to the highly conductive phase is small, and the model practically does not differ from the ordinary Laplace model with a single, highly conductive, phase.

The two-stage model of plasma behaviour (transition of a streamer to a highly conductive phase) describes the internal structure of the growing tree in more detail than the

ordinary Laplace models, in which the conducting structure was assumed to be equipotential. On the other hand, our model is simple enough in comparison with the models of [13–15], in which attempts were made to describe charge relaxation directly using Ohm's law and Poisson's equation.

The macroscopic model with the SL transition can be useful for computer simulations of this phenomenon and is very promising for applications in technology. It is also very interesting to use this approach in models based on Poisson's and charge transfer equations.

Acknowledgments

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