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Stochastic Features of Initiation of Liquid Dielectric Breakdown at Small Area of Positive Electrode

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Abstract

The regularities of breakdown initiation in dielectric liquids in uniform and quasi-uniform pulse electric fields are studied in terms of the function $\mu(E)$ introduced in [1]. This function means the probability density of a streamer inception at a small element of an electrode area in a short time interval. The dependence of the function $\mu(E)$ on an electric field is reconstructed from experimental data on a breakdown in *n*-hexane [2]. The increase in the pulse electric strength of dielectric liquids in narrow gaps for hemispherical electrodes is analytically explained.

Introduction

It is known that the prebreakdown processes in liquid dielectrics proceed in two stages. The first of them is the development of a series of microscopic processes at the electrode surface and in a thin layer of the dielectric contiguous to it. These processes result in appearance of one or several luminous sprouts at the surface of the electrode. These regions of a new phase are able to conduct an electric current. The duration of the first stage (called statistical time lag) is a random quantity for which the probability density depends on the electric field and its distribution along the surface of the electrode. At the second stage, a conductive branching structure (called streamer) grows rapidly from these sprouts deep into the interelectrode gap.

Numerous experimental data point to the principal role of stochastic processes at a breakdown in dielectric liquids (for example, statistical time lag, asymmetry and non-reproducibility of streamer detailed structure, tooth-like shape of recordings of current and light pulses, etc.). Thus, a correct description of stochastic regularities of a breakdown is required to develop an adequate model for this phenomenon.

To describe the essentially stochastic character of prebreakdown processes in dielectric liquids it was proposed to use the distribution function of statistical time lags $f(E)$ [3]. This function means the probability density of breakdown initiation in a short time interval. In 1993 it was proposed that the basic stochastic processes of streamer inception at the electrode be described by the function $\mu(E)$ [1]. This function means the probability of streamer inception in a short time interval at a small element of an electrode surface near which the electric-field value equals E. The function $\mu(E)$ depends on the properties of the dielectric being investigated and, probably, on the material of the electrode. The function $\mu(E)$ increases sharply with increase in the electric field. It is obvious that $f(E) = \int \mu(E)\, ds$, where the domain of integration is the entire electrode area.

The prebreakdown processes occur by the thermal, bubble-like or ionization mechanisms and begin at the anode or cathode, depending on pressure and electric field. In [2,4,5] the breakdown of *n*-hexane in strong electric fields $E_0 > 1$ MV/cm in the time range from 10 to 1000 ns was investigated. At normal pressures there was a competition between the bubble-like mechanisms of breakdown both from the cathode and from the anode and the ionization mechanism of breakdown from the anode. As the pressure increased to ~1 MPa, the bubble-like mechanisms were suppressed.

In [6,7] the field fluctuation criterion (FFC) of inception and growth of streamers was proposed, which actually involves the dependence $\mu(E)$ in the form

$$\mu(E) = \frac{1}{s_0 \tau_0} \exp\left(- \frac{E_{*0} - E}{g} \right).$$

It was shown that results of streamer growth simulations using the FFC qualitatively describe the basic stochastic regularities of breakdown obtained experimentally [1,6]. A similar form of dependence $f(E) = \exp(\alpha E - C)$ was used for approximation of experimental data on the probability density of a breakdown in *n*-hexane for rectangular voltage pulses [3]. Physically, the exponential dependence corresponds to ionization processes such as thermal-electric ionization and dissociation.

For any dielectric liquid being investigated the function $\mu(E)$ can be reconstructed from experimental data obtained under breakdown conditions where one or another mechanism of breakdown prevails. It is important in addition to verify the main hypothesis that rather than calculate the complex processes at a microscopic level, one can use the probability function $\mu(E)$ to describe stochastic processes of breakdown inception quantitatively.

Statistical time lags of breakdown

Generally, besides the statistical time lag the measured time lags of a breakdown also include the formative time of a streamer. To obtain histograms of statistical time lags distribution, it is necessary to measure moments of streamer initiation, and this is rather com-

plicated. Instead, narrow gaps ~ 100 μm are usually used, for which the formative time is much less than the statistical time lag [2,3].

It has been shown previously that the distribution function of statistical time lags in strong electric fields has the form [1]

$$\varphi(t) = \left(\int_S \mu(E)\, ds \right) \cdot \exp\left(-t \int_S \mu(E)\, ds \right), \quad (1)$$

where integration should be carried out over the entire electrode area. In the derivation of this distribution it was assumed in particular, that the electrode area is much greater than the characteristic scale of microprocesses, that lead to the formation of a streamer, and that the initiation probability does not depend on preceding instants of time. The latter, for example, is not true when there is noticeable charge injection from the cathode. Thus, the mean statistical time lag is expressed through the integral

$$\langle t \rangle = 1 \Big/ \int_S \mu(E)\, ds \ .$$

For given geometry of an interelectrode gap, it is possible to find the dependence μ(E) from a set of experimental data on applied voltage dependence of $\langle t \rangle$. In the simplest case of a flat electrode system $\mu(E) = \left(\langle t \rangle \cdot S \right)^{-1}$, where S is the area of an electrode. Otherwise, it is necessary to calculate the electric-field distribution along the surface for each configuration of the electrodes used in experiments, and then to solve the inverse problem of reconstructing the integrand function μ(E). Unfortunately, the experimental data on a breakdown in dielectric liquids available in the literature are, as a rule, obtained for flat electrodes of finite size, for which it is impossible to neglect the edge effect, or for more complex configurations, such as coaxial cylinders, hemispherical electrodes or point-plane geometry.

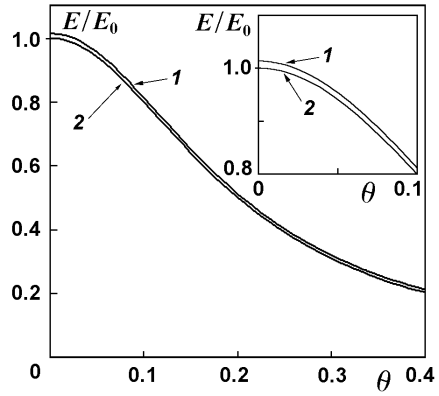

Fig. 1. Electric field distribution along the surface of a spherical electrode for β = 0.02.

Calculation of the electric-field distribution between hemispherical electrodes

For spherical electrodes of radius R, used at separation between them d, the electric field was obtained analytically by solving the Laplace equation in bispherical coordinates in the region of the interelectrode gap. In a Fig. 1 the plot of a relative electric field E/E_0 at electrode surface is given depending on a polar angle θ on a sphere (curve *1*). The direction $\theta = 0$ is correspond to the maximum field along a symmetry axis. Here $E_0 = V/d$ is an average electric strength along an axis between electrodes, V is applied voltage, and $\beta = d/2R$ is a relative quantity of a gap between electrodes.

For a quasi-uniform field in a narrow gap between spherical electrodes the following approximate formula is valid:

$$E \approx \frac{E_0}{1 + (1 - \cos\theta)/\beta} \ . \quad (2)$$

Only a small part of the electrode area near the symmetry axis makes a major contribution to breakdown inception because of the sharp dependence on the field μ(E). For this region, the approximate formula (curve *2*) practically coincides with the exact solution. For example, even for a gap length of 200 μm, the difference is less than 2% of the maximum field value.

Reconstruction of μ(E) from experimental data

From formula (2) for narrow gaps it was obtained [1]

$$\frac{1}{\langle t \rangle} = \int_S \mu(E)\, ds \ \approx \ d\pi R E_0 \int_0^{E_0} \frac{\mu(E)}{E^2} dE \ . \quad (3)$$

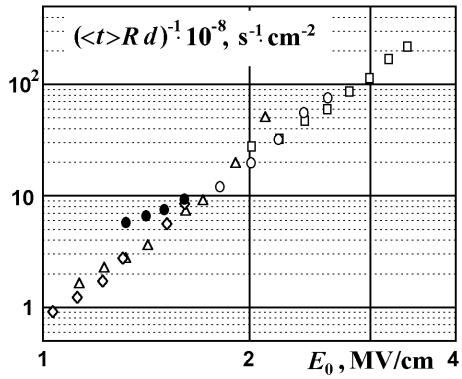

Fig. 2. Dependence of quantity $< t > Rd$ on E_0. d=25(□), 50(o,●), 100(Δ), 150(◊) μm.

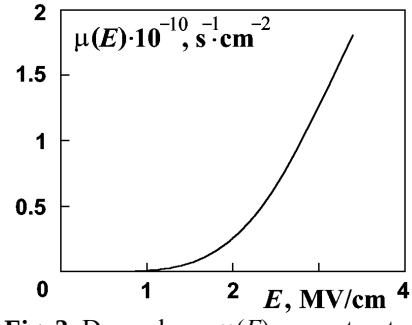

Fig. 3. Dependence μ(E) reconstructed from experimental data [2].

It turned out that the quantity $< t > Rd$ depends only on E_0. In fact, all results of experiments on breakdown in *n*-hexane for narrow gaps [2] are located near a certain line (Fig. 2). This indicates that the integrand function μ(E) can be obtained from experimental aver-

age values of statistical time lags $< t_i >$ for various applied voltages and gaps. The experimental data for d = 50 μm, marked by black color, were rejected since they contradict those obtained for d = 100 μm (Fig. 4).

This inverse problem was solved using the method of regularization [8]. The function $\mu(E)$ was reconstructed by minimization of the functional

$$F[\mu(E)] = \sum_{i=1}^{N} \left(\int_S \mu(E_i(s))\, ds - \frac{1}{\langle t_i \rangle} \right)^2 + \alpha \int_0^{E^*} \left(\frac{d^2\mu}{dE^2} \right)^2 dE .$$

where N is number of experimental data for $< t_i >$, α is the regularization parameter, and E^* is the maximum electric-field strength over the entire set of experimental data used. The dependence on the electric field $\mu(E)$ was sought in the class of monotonically increasing functions ($d\mu/dE > 0$). Thus, it is possible to find a function $\mu(E)$ that fits the set of experiments best of all by solving the problem of minimization of this functional $F[\mu(E)]$. A grid method that represents F as a function of a set of variables $\mu_k(E_k)$ was used. A minimum of this function of many variables was sought using a numerical algorithm based on the Monte-Carlo method.

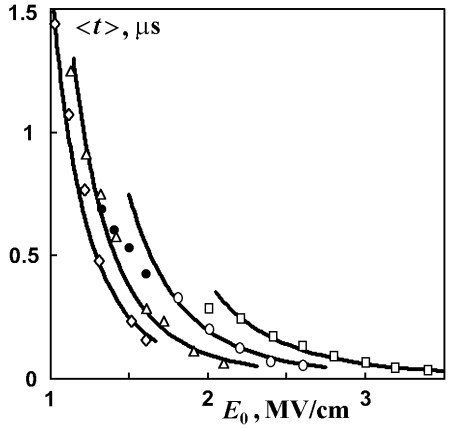

Fig. 4. Average statistical time lags of a breakdown $< t >$ versus E_0. Experimental data [2] and analytical dependencies obtained from (3) and (4) for d = 25(□), 50(o,●), 100(Δ), 150(◊) μm.

Results

Experimental data [2] on breakdown of *n*-hexane in narrow gaps between hemispherical electrodes were processed. These experiments were performed in the range of E_0 from 1 to 3.5 MV/cm at a pressure P_0 = 10^5 Pa. Hemispherical stainless-steel electrodes of radius R = 0.5 cm were used. Figure 3 shows the dependence $\mu(E)$ that describes the set of experimental data used most adequately. As expected, the probability of a breakdown sharply increases with increase in the electric field. At the same time, formula (3) is in good agreement with these experimental results (Fig. 4) when the following power dependence is used:

$$\mu(E) = A E^n . \qquad (4)$$

Here n = 4.65 and $A \approx 9.4\ 10^7$ cm^{-2} s^{-1} (MV/cm)$^{-4.65}$.

This dependence is even somewhat more convenient for engineering applications.

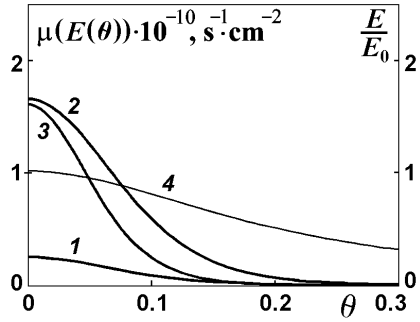

Fig. 5.

Using the function $\mu(E)$ obtained it is possible to calculate a number of dependencies of the breakdown probability under a pulse voltage in *n*-hexane for various geometry of electrodes, magnitude, duration, and shape of the voltage pulse. Figure 5 gives the probability density distribution of breakdown initiation $\mu(E(\theta))$ along the surface of a spherical electrode. Curve *1* was obtained for E_0 = 2 MV/cm and β = 0.02, and curves *2* and *3* were obtained for E_0 = 3 MV/cm, β = 0.02 and 0.01, respectively. The exact solution for the electric-field distribution for β = 0.02 is shown by curve *4*.

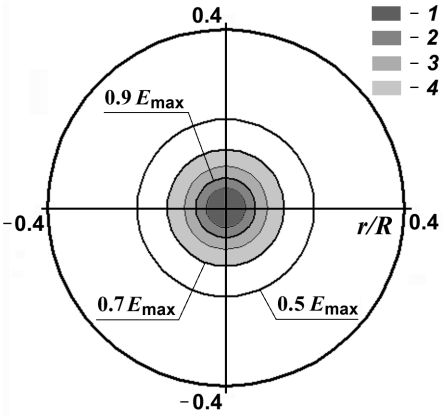

Fig. 6. Distribution of places of breakdown initiation on a spherical electrode surface.

Figure 6 gives the density function of places of breakdown initiation on the surface of a spherical electrode for β = 0.02. The regions where $\mu/\mu_{max} > 0.8$, 0.6, 0.4, and 0.2 are shown (gray levels *1*, *2*, *3*, and *4*, respectively). These results are in agreement with experimental data obtained in [9] at a pulse voltage. The region of noticeable breakdown probability decreases considerably for narrow gaps (Fig. 5, curves *2* and *3*).

Figure 7 shows the probability of breakdown inception

$$p(E) = 1 - \exp(-\mu(E)\, S\, \tau), \qquad (5)$$

obtained from (1) for a rectangular voltage pulse at $S\tau$ = 0.01, 0.001, and 0.0001 cm^2·μs (curves *1*, *2*, and *3*, respectively) in the case of flat electrodes. Here S is the electrode area and τ is the pulse duration.

From (4) and (5) it is easy to find the electric field

$$E_b = \left(\ln(1/(1-p))/(A\, S\, \tau)\right)^{0.215} ,$$

which corresponds to the fixed probability p (Fig. 8).

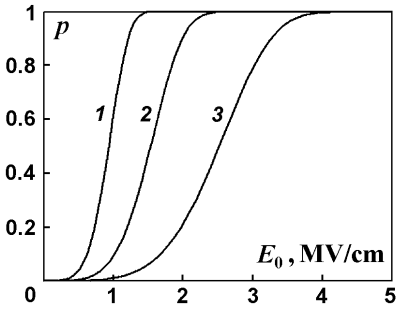

Fig. 7. Probability of breakdown inception for flat electrodes.

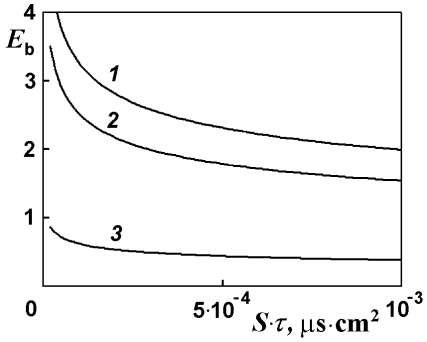

Fig. 8. Breakdown stress E_b vs. product $S\,\tau$ for $p = 0.9$, 0.5, and 0.001 (curves *1*, *2*, and *3*, respectively).

Discussion and conclusions

Under pulse voltage, the electric strength is known to increase as the gap length between spherical electrodes decreases. This is mainly because the geometrical size of a region in which the electric field is close to E_0 diminishes according to formula (3). The second reason is that the maximum field E_{max} on the electrode surface (for $\theta = 0$) is greater than E_0 and tends to E_0 as d decreases. Hence, the value of μ_{max} decreases with the gap length for E_0 = const (Fig. 5, curves *2* and *3*).

Unfortunately, because of the roughness of electrode surface only a certain effective dependence of μ on electric field was actually found by our method. This dependence is correct only for fixed electrode roughness. However, the roughness scale is usually insignificant ($\delta \sim 1$ μm $\ll d$), and, hence, the field enhancement does not depend on the gap length and is practically constant at relevant points of the surface. As it turned out, the dependence $\mu(E)$ obtained is close to a power law for an electric field of ~ 1 MV/cm. In this case, it is possible to obtain an estimate of the influence of the electrode roughness. The true values of $\mu_0(E) = (A/\lambda)\,E^n$ are λ times smaller than the function $\mu(E)$ that we found. The quantity $\lambda > 1$ and can reach values of ~ 100 depending on the quality of the electrodes surface but it does not depend on E. In fact, the roughness influences only the coefficient A but does not the exponent n. Hence, the form of the dependence does not change.

The form of the function $\mu(E)$ depends on the physical mechanism of the ionization processes leading to generation of a conductive phase, and only the parameters of this function depend on the dielectric being investigated and the electrode material. It is of great interest to elucidate the physical mechanisms of breakdown of the dielectric liquid itself. To investigate the purely ionization mechanism of breakdown initiation at the anode, it is necessary to have a sufficient body of reliable experimental data. These data on average statistical time lags should be obtained for strong electric fields $E_0 > 1$ MV/cm (range of time from 10 to 1000 ns) at elevated pressure $P > 1$ MPa at which the thermal and bubble-like mechanisms would be suppressed. The method described above allows one to reconstruct the function $\mu(E)$ even in the case where the Weibull plots usually used [9] are not straight lines, for example, for an exponential dependence.

The function $r(E)$ used in stochastic models of streamer growth has a similar physical meaning. The growth rate of streamer branches is proportional to this function [10]. Hence, information on the function $\mu(E)$ can be useful for clarifying the form of the streamer growth rate dependence on the electric field before the streamer tips. Thus, the results obtained should be taken into account in stochastic criteria for the inception and growth of streamers [11] in simulations of a breakdown in dielectric liquids.

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