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Simulation of the Local Electric Field at the Tips of a Growing Streamer at the Breakdown in Liquid Dielectric

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***Abstract*— The simulations of the growth of branching streamer structures were performed for the electrode gap of the length up to 1 mm in a point - plane geometry. The stochastic model of growth developed earlier was used. The parallel algorithm for Graphic Processing Units (GPU) was specially designed for high-performance computations of the electric field, the charge transfer and the stochastic growth. This allowed us to perform the simulations on very large lattices with the sizes not less than 400×400×400 nodes for the time intervals of order of 1 microsecond. The simulations were performed at the realistic physical scales in space and in time for the first time. The estimations of the electric field strength at the tips of the growing streamer as well as the average conductivity of streamer channels were made for the streamers with velocities about 5 km/s. The electric field at the streamer tips of about 20 MV/cm was approximately constant during the growth.**

Keywords—streamer; stochastic growth; electric field strength; electric current

I. INTRODUCTION

The key question for understanding the mechanism of inception of streamer in dielectric liquid is the question about the magnitude of the electric field strength at the tip of the streamer channel. Plenty of experiments were carried out with high spatial resolution for streamer structures of different types [1, 2]. Nevertheless, the value of the electric field strength necessary for the onset of the streamer channel formation is still unknown.

From one hand, the radius of the channel tip is of the order of 1 μm that requires a high spatial resolution of the equipment registering the processes at the streamer tips. From the other hand, the size of the region where the streamer structure develops (as well as the size of the structure itself) is two or three orders of magnitude larger than the average channel diameter. The way along which the streamer tip propagates in liquid dielectric during its growth is unpredictable because of the stochastic nature of the process. Hence, the direct measurements of streamer characteristics are complicated.

There are unsolved problems also at the mathematical simulation of the streamer growth. The streamer structure provides the charge transfer into the electrode gap in the case of discharge from the electrodes or the redistribution of the charges in the gap in the case of non-electrode streamers [3]. This leads to the essential change of the field configuration between the electrodes [4, 5] that influences the streamer characteristics such as the growth velocity. The exact calculation of the ionization processes in a liquid with immersed bubbles and in the gaseous channels in liquid is of great interest [6]. Unfortunately, this approach does not allow taking into account the branching of the channels and the influence of them on each other at present. Moreover, these models do not include the description of hydrodynamic processes that lowers the value of the results obtained.

Most simulations with the models of stochastic growth of branching streamer structures [7–10] were performed on lattices with a rough spatial resolution because of the lack of the computational resources. In those simulations, the fine spatial structure of the streamer channels can not be represented with the real spatial resolution. The change of the field strength ahead of the channel tips during the growth was described only qualitatively. We obtained very rough estimation of the electric field magnitudes ahead the tips. The hydrodynamic expansion of the segments of the streamer channels could be approximated only with the rough model.

In the present work, the model of the growth of branching streamer structures [7, 8] was realized on the spatial lattice with sufficiently high resolution (~ 2.5 μm) that allowed resolving the fine structure of streamers. The electric field distribution is recalculated for the entire electrode gap according to the charge distribution in the gap at the moment. The original software was developed for the parallel computations with the use of NVIDIA graphic cards and CUDA programming technology. The parameters of the model were chosen by the comparison of the results of simulations with the known experimental data on the development of the streamers during a pulse breakdown in hydrocarbon liquids [1]. This allowed us to obtain the most realistic estimation for the local electric field strength in the vicinity of the tips of the growing streamer channels.

II. THE MODEL

The model of stochastic growth of streamer structure developed in [7, 8] was used. This model assumes that the

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probability of the formation of the conducting channel of the streamer structure is the function only of the local electric field strength $\mathbf{E}$.

Streamer growth was simulated as a discrete in space and in time process of joining of new conducting elements to the previously formed conductive tree (or to the electrode at the first time step) in accordance with simple probabilistic rules. The probability of the formation of a new element of a length l during the time step of the duration τ obeys the law of rare events [11, 12] and it is equal to

$$p(E,\tau) = A \cdot \exp(E/g) \cdot \tau / l , \quad (1)$$

that is valid for very short time steps τ. Here g is the characteristic scale of the fluctuations of the local electric field that leads to the local breakdown in a dielectric, A is a constant. The probability density distribution of the new element formation in time is

$$f(t) = r(E) \cdot \exp(-r(E) \cdot t) , \quad (2)$$

where $r(E) = -\ln(1 - A\tau\exp(E/g)/l)/\tau$. For this case, the local velocity of the channel tip can be calculated with the following formula [11, 12]

$$\upsilon(E) = A \cdot \exp(E/g) . \quad (3)$$

This formula is considered as an approximation of the real dependence of the velocity of the streamer tip on the local electric field strength in some range of the values of E. The average velocity of the streamer structure is related to this local velocity of an individual channel by a non-linear way. Generally speaking, we can find the parameters of the model A and g comparing the velocity of the streamer structure in the simulations with the velocity in the experiments on pulse electric breakdown at different values of the applied voltage. Note that the expression (2) has the form that is usually used to describe the time-voltage characteristics of the breakdown in semi-uniform geometry provided that $r(E)$ is a power function.

We simulate the growth of the streamer on the cubic lattice with the spatial step h. New conducting element of the structure was simulated with a linear segment connecting two neighbor nodes of the lattice. Each node of the lattice has 26 neighbor nodes. Thus, three types of the elements of the lengths $l_1 = h$, $l_2 = \sqrt{2}h$, and $l_3 = \sqrt{3}h$ formed the streamer structure in the simulations. The ends of the conducting segments belonged to the streamer structure in contrast to others nodes that represented the dielectric.

The probabilities of the appearance of the new segments (1) were calculated using the following algorithm. Let us start from a node that belongs to the streamer structure. Find the value of the projection of the electric field E_i onto each of the possible direction to the neighbor nodes that belong to the dielectric state. If the Field Fluctuation Criterion (FFC) [7]

$$E_i > E_* - \delta_i , \quad (4)$$

was fulfilled then the segment between these nodes became the new element of the conducting structure. The distribution $\varphi(\delta) = \exp(-\delta/g)/g$ was used for the probabilities of the fluctuations δ that was equivalent to the random variable $\delta = -g\ln(\xi)$. Here, ξ is a random value uniformly distributed within the interval from 0 to 1. The parameter was $E_* = -g \cdot \ln(A \cdot \tau / h)$.

III. THE ELECTRODYNAMIC EQUATIONS

The electric field and the charge distribution in the gap between electrodes were calculated at each time step by solving the following system of equations

$$\mathrm{div}(\varepsilon\nabla\varphi) = 0 , \quad (5a)$$

in the region occupied by dielectric, and

$$\mathrm{div}(\varepsilon\nabla\varphi) = -4\pi\rho , \quad (5b)$$

in the region occupied by a streamer structure. Here ε is dielectric permittivity of a substance. Thus, we do not take into account the injection of electric charges into liquid from streamer channels.

For the conductive streamer channels we have the equations

$$\frac{\partial\rho}{\partial t} = -\,\mathrm{div}\,\mathbf{j} , \quad \mathbf{j} = \sigma \cdot \mathbf{E} , \quad \mathbf{E} = -\nabla\varphi . \quad (6)$$

where φ and $\mathbf{E}$ are the electric field potential and the electric strength, respectively, ε is the dielectric permittivity, ρ is the electric charge density, σ is the average conductivity of the streamer channels that, generally, depends on the electric field strength in channel, $\mathbf{j}$ is the current density in the channels.

The equations (5) and (6) were solved at each time step with the use of the conservative implicit in time finite-difference scheme developed earlier [8]. At each time step, the distribution of the potential was calculated by simple iterations. The calculation of the electric field distribution takes more than 90 percents of the simulation time. This is the main restriction on the size of the lattice since the time of calculations increases with the number of the lattice nodes.

The previous calculations [8, 9, 11, 12] were performed on the lattices of the relatively small size (up to 60×60×60 nodes).

IV. REALIZATION ON THE GRAPHIC PROCESSING UNIT

The numerical method [8] was realized on NVIDIA Graphic Processing Units (GPU) for calculating the currents and the electric potential from the system (5) and (6). This system of equations was solved iteratively at each time step after the procedure of choice of new conducting elements.

The parallel algorithm for the growth of the streamer structure was specially designed. Each node of dielectric adjacent to the existing conducting structure or to the electrode was examined according to the FFC. If the criterion (4) was fulfilled for some segment connecting this node to the node of the conductive structure, this node was added to the streamer structure.

The CUDA programming technology with C language was used to implement the algorithm on GPU. The graphic card with 512 processor cores was used in simulations. Each lattice node was handled in its own thread. The blocks of 32 threads

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provided the maximum computing performance. The use of GPU accelerated the calculations by 100 times. All the data (electric field potential arrays, charges array, array of the node states, etc.) were allocated in the fast global memory of GPU.

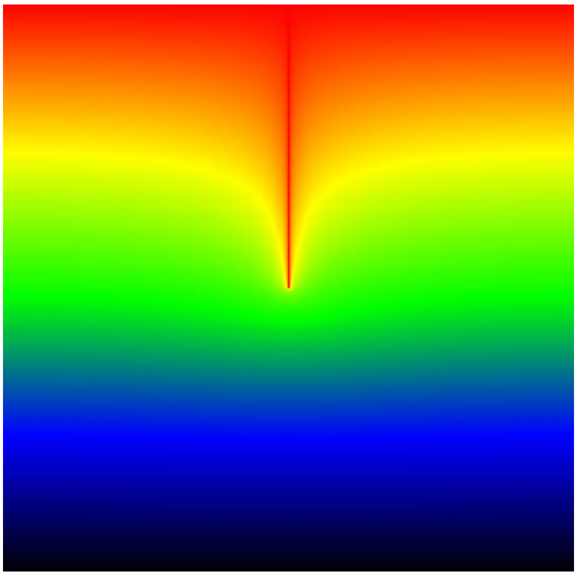

Fig. 1. Distribution of the electric field potential in the central cross section of the gap calculated with GPU. Lattice size is 450×450×450.

The parallel computations allowed us to carry out the three-dimensional simulations on the lattices larger than 400×400×400 nodes.

Fig. 1 shows the potential of the electric field in the cross-section of the electrode gap calculated on the lattice of the size of 450×450×450 nodes. The length of the protrusion is 225 lattice steps. The coefficient of non-uniformity of the electric field is 32. The calculations of the potential were made with the relative accuracy better than 10^{-8}.

V. RESULTS OF SIMULATIONS

Simulations of streamer growth were carried out for the case of point – plane electrode geometry on the cubic lattice of size of 386×386×386. Length of protrusion imitating point electrode was 192 lattice steps with the tip radius of 2.6 μm that corresponds to the coefficient of the field non-uniformity 33. The distance from the tip of the protrusion to the opposite plane electrode was 192 lattice steps and corresponded to the gap length of 0.5 mm. The time step was equal to $\tau = 10$ ps.

The parameters of the model were chosen after the comparison of the simulation results with the experimental data on the development of the streamers at the pulse breakdown in liquid hydrocarbons [1]. We used the velocity of the streamer structure and its geometrical form to define the values of A and g. These values cannot be determined unambiguously because there are no clear geometrical characteristics of the streamer form. Nevertheless, there is visual distinction between the bush-like and tree-like streamers. The geometrical form of the streamer structure in our simulations varied significantly with small variations of the coefficient g. The other parameter A influences mainly the value of the streamer average growth velocity. We obtained the values of the growth parameter $g = 0.7$ MV/cm and the coefficient $A = 1.4\ 10^{-7}$ m/s before the exponent in the formula (3).

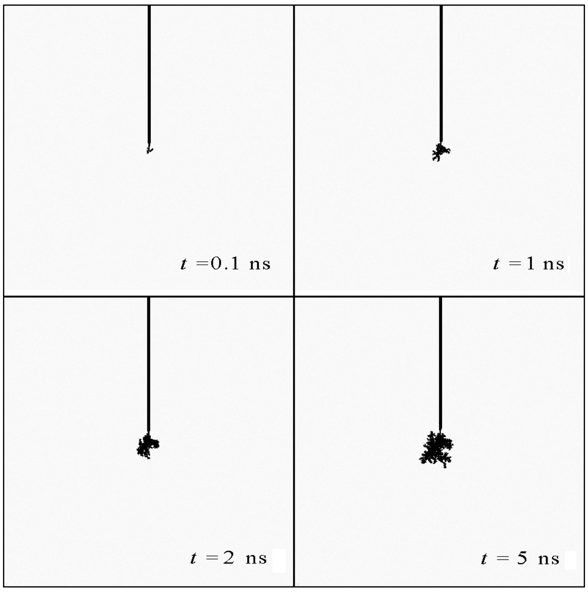

Fig. 2. Initial stages of the growth of the streamer structure. V = 30 kV, d = 0.5 mm. Lattice size is 386×386×386.

Note that the velocity (3) of a single channel is of order of one hundred kilometers per second if the local electric field strength is slightly below 20 MV/cm. This value of the velocity is by an order of magnitude higher than the average velocity of the propagation of the whole streamer.

The last parameter that governs the growth of the streamer in our model is the conductivity of the channels σ. We used the experimental recordings of the current in the external circuit during streamer growth to set the magnitude of the current pulses we observed at the simulations. The magnitude of the current depends mainly on the conduction of the channels.

We neglected the hydrodynamic expansion of the channels considering that the channel radius was equal to 2.6 μm. In this case, the conductivity of the channels averaged across the section was calculated as $\sigma = 3.8$ S/m.

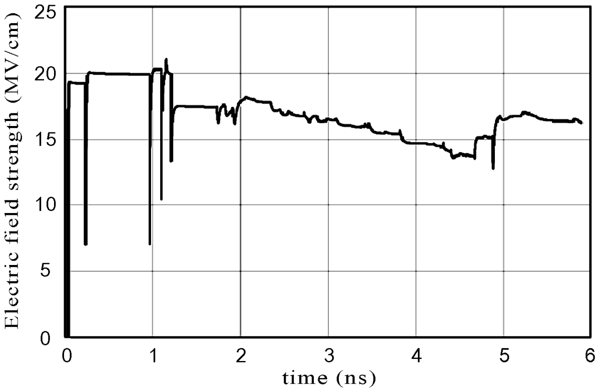

Fig. 3. The maximal values of the local electric field strength at the tips of the growing streamer. V = 30 kV, d = 0.5 mm.

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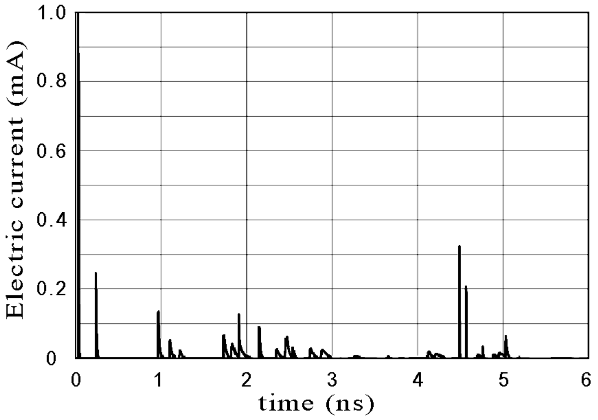

Fig. 4. The pulses of the electric current accompanying the streamer growth in the external curcuit. $V = 30$ kV, $d = 0.5$ mm.

Fig. 2 shows the shape of the streamer structure in our simulations at the initial stages of the growth. The average velocity of the structure growth was calculated by the measurement of the distance from the point electrode to the most distant tip and was about 5 km/s.

The growth was larger near the point electrode and decreased significantly as the streamer propagated to the central part of the electrode gap.

Fig. 3 shows the maximal values of the local electric field strength ahead the tips of the streamer structure at different moments of time. The maximal values are shown over all the tips of the structure at the current time step. The maximal electric field strength decreased as the streamer advanced. The electric field was about 5 MV/cm at the tip of new conducting element of the streamer structure and it rose up to about 20 MV/cm due to the charge relaxation in the channels.

The electric current induced by the growth of the streamer structure is shown in Fig. 4. When the structure made the next step deep into the gap the new pulse of electric current was observed with very short front and the tail of the duration of about 0.1 ns. The appearance of a new element in the region of dielectric inside the existing structure (between the branches) was accompanied by the current pulses of small magnitude. Earlier, we proposed the model of “pulse” conductivity of the streamer channels that allowed simulating the waves of conductivity in the streamers [9]. These waves simulated the partial discharges in the streamer channels observed in the experiments as the waves of the luminescence of the discharge plasma. Each current pulse in the circuit corresponded to one flash of the streamer during partial discharge. Single pulses merged giving a quasi-continuous conduction current from time to time. Each wave of conductivity was initiated by the formation of new element of the streamer in most cases. Nevertheless, the streamer structure became non-conductive after the discharge wave had decayed. The conduction current was zero at these periods of time. The similar situation is observed in the present work. Single pulses of the current in Fig. 4 appeared due to the step propagation of the streamer structure. In contrast to the work [9], the conduction current in the structure in Fig. 2 was continuous but it had very small magnitude during time between the pulses.

VI. CONCLUSION

The model of streamer growth was realized on the large spatial lattice that allowed us to resolve the spatial structure of the streamer in the simulations. The simulations of the streamer growth were carried out for the lattice with mesh step 2.6 µm that is of the order of the initial diameter of the streamer channel. This allowed us to make the estimations of the local electric field strength E at the tip of the streamer channels. The values E from 15 to 20 MV/cm was obtained.

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