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Stochastic Model of Breakdown Initiation in Dielectric Liquids under AC Voltage

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***Abstract*:** An electric strength of perfluorodibutyl ether and transformer oil in small gaps between the hemispherical electrodes was investigated experimentally under AC voltage of linearly increasing amplitude. The influence of radius of hemispherical electrodes, gap lengths and also rate of voltage increase on probability of breakdown initiation was investigated. In the macroscopic stochastic approach proposed in [1-4], breakdown initiation is described by macroscopic function $\mu(E)$. This function is a probability density of a streamer initiation. The dependences of breakdown initiation probability on various parameters of experiment mentioned above were obtained for flat, cylindrical and hemispherical electrodes under DC and AC voltage of linearly increasing amplitude. The method of fixed breakdown probability was developed to reconstruct the function μ(E) in the case of non power-law approximation using experimental data on series of breakdown voltages. The values of function $\mu(E)$ for transformer oil and perfluorodibutyl ether were determined in the range of electric field 0.3–1.0 MV/cm. Stochastic computer simulations of breakdown inception were carried out.

INTRODUCTION

It is usually implied that ability of a dielectric to maintain the dielectric properties under the action of strong electric fields is characterized by its electric strength. However, it is well known, that average value of electric field, at which breakdown of a dielectric occurs, also depends on specific experimental conditions such as the form and the sizes of electrodes, distance between them, magnitude and the form of applied voltage, etc [5-9]. Therefore, the classical concept about fixed "electric strength" fails. Instead of this, the concept of "dynamic electric strength" of dielectric that depends on the specific conditions listed above have to be used. Well known time-voltage curves are the particular feature of dynamic electric strength.

Moreover, it is well known that the prebreakdown processes in liquid dielectrics have a stochastic nature. Numerous experimental data point to the principal role of stochastic processes at a breakdown in dielectric liquids. Thus, an adequate description of stochastic regularities of dielectric breakdown has to include probability distribution functions for such processes.

One of the stochastic processes is the initiation of breakdown due to the development of a series of microscopic phenomena at the electrode surface and in a thin dielectric layer contiguous to it. The duration of this stage is a random value for which the probability density depends on the electric field and its distribution along the surface of the electrodes.

Many authors made efforts to describe stochastic regularities of breakdown using various statistical distributions [5, 6, 9-11]. Unfortunately, these approaches do not allow one to describe in a simple way how the complete set of experimental conditions (duration and waveform of applied voltage, form and size of electrodes, etc) influences the breakdown.

In [1-2] it was proposed that the basic stochastic processes of streamer inception at the electrode could be described by macroscopic function $\mu(E)$. This function is the probability density of breakdown initiation in a short time interval at a small element of an electrode surface near which the electric-field value equals to E. The function $\mu(E)$ increases sharply with increase in the electric field. The parameters of the function $\mu(E)$ depend on the properties of a specific dielectric and on the properties of the electrodes. The function $\mu(E)$ directly defines also the dynamic electric strength for every specific experimental condition.

This macroscopic approach allows one to obtain the dependences of the breakdown initiation probability in time on the applied voltage, its waveform, electrode area, and gap length and to simulate the breakdown, including its stochastic features. And vice versa, it is possible to reconstruct the function $\mu(E)$ from experimental data. The conception of dynamic electric strength follows from stochastic approach by averaging the breakdown voltages over the probability distributions.

EXPERIMENTS

The experiments on breakdown in synthetic transformer oil "TECHNOL 2002 (ISO 9001)" were carried out. A new pair of polished spherical stainless steel electrodes with surface radius $R = 19$ mm were used in each series of experiments. The gap lengths between the electrodes d were in the range from 0.5 to 2.5 mm. The amplitude of AC voltage of frequency 50 Hz increased with a constant rate.

In experiments, the current effective value of voltage V_{EFF} at which breakdown of a dielectric occurred was registered (Fig. 1). A built-in electronic device removed the voltage from the electrodes immediately after breakdown. The rate of increase of effective value of applied voltage k_e = 0.5, 1, 3 kV/s was switched over after each breakdown. Thus, three data sets of breakdown voltages were obtained in one series of experiments under identical conditions. Period between breakdowns was approximately 3 minutes. The conditioning effect was observed in every series of breakdowns. We took into account only the breakdowns after first 45 shots in series. These results are shown in the Table I.

TABLE I.

N	d, mm	k_e, kV/s	N_0	$<V_{EFF}>$, kV	$<E_0>$, kV/cm	V^*_{EFF}, kV	E^*_0, kV/cm
	2.5	0.5	60	50.6	286	53	300
1	2.5	1	60	55.5	314	58	328
	2.5	3	60	64.0	362	71	402
	1.0	0.5	40	24.1	341	25	354
2	1.0	1	40	24.6	348	25.5	361
	1.0	3	39	29.8	421	34	481
	0.5	0.5	48	20.5	580	21.5	608
3	0.5	1	50	22.4	634	24	679
	0.5	3	50	23.8	673	25	707
	0.83	0.5	27	25.7	438	30	511
4	0.83	1	27	29.1	496	31	528
	0.83	3	27	27.6	470	30.5	519
	1.66	0.5	25	32.6	278	36	307
5	1.66	1	25	38.9	331	42	358
	1.66	3	25	45.8	390	49	417
	2.5	0.5	25	42.2	238	46	260
6	2.5	1	25	46.7	264	49	277
	2.5	3	25	57.0	322	61	345

Here N is the series number, N_0 is the number of breakdowns, $<V_{EFF}>$ is the average effective value of breakdown voltage, $<E_0>$ is the corresponding value of amplitude of electric field averaged along an axis between electrodes, V^*_{EFF} is the effective value of a voltage at which breakdown occurred with the probability $P_+(t) = 0.63$, E^*_0 is the corresponding amplitude of electric field averaged along an axis of electrodes.

The experiments on breakdown in the perfluorodibutyl ether were also carried out (Table II). The liquid was previously boiled for degassing with a backflow condenser to prevent boiling out of liquid and then was filtered. The effective value of AC voltage increased with a constant rate $k_e = 2$ kV/s. The surfaces of hemispherical stainless steel electrodes were polished before each series of experiments.

DISTRIBUTION OF THE ELECTRIC FIELD

A good approximation for electric field strength on the surface of hemispherical electrodes was obtained analytically by solving the Laplace equation using bispherical coordinates in the gap between two metallic spheres:

$$E(\xi,\eta) = \frac{E_0 d\sqrt{2(\mathrm{ch}\xi - \cos\eta)}}{2R\,\mathrm{sh}\xi_1} \sum_{l=0}^{\infty} \frac{e^{-(l+1/2)\xi_1}\, \mathrm{P}_l(\cos\eta)}{\mathrm{sh}((2l+1)\xi_1)} \times \qquad (1)$$

$$\left\{\mathrm{sh}\left(\left(l+\frac{1}{2}\right)(\xi+\xi_1)\right)\mathrm{sh}\xi + 2(\mathrm{ch}\xi - \cos\eta)\left(l+\frac{1}{2}\right)\mathrm{ch}\left(\left(l+\frac{1}{2}\right)(\xi+\xi_1)\right)\right\}.$$

Here $E_0 = V/d$ is an average electric field strength along an axis between electrodes, V is the applied voltage, R is the radius of spherical electrodes, and d is the gap length between them, ξ and η are bispherical coordinates (electric field does not depend on azimuthal angle α because of axial symmetry of the problem, $-\xi_1 < \xi < \xi_1$, $0 < \eta < \pi$), P_l is the Legendre polynomial of index l, $\xi_1 = \ln(1+\beta+\sqrt{\beta(2+\beta)})$, $\beta = d/2R$ is the reduced length of the gap. The relation between bispherical

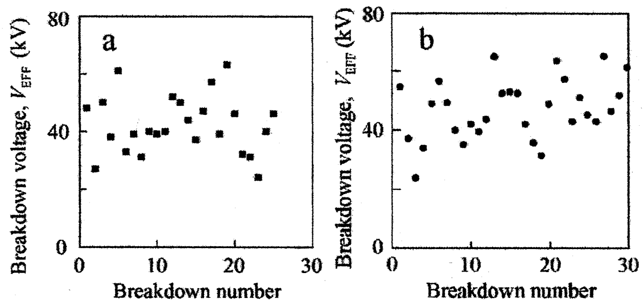

Fig. 1. Typical series of breakdowns in transformer oil under AC voltage of linearly increasing amplitude. Experiment (a) and computer simulation (b). $d = 2.5$ mm, $k_e = 0.5$ kV/s.

coordinate η and polar angle θ on the sphere is given by the expression $\cos\eta = (1-(1+\beta)\cos\theta)/(1+\beta-\cos\theta)$.

TABLE II.

d, mm	R, mm	N_0	$<V_{EFF}>$, kV	V^*_{EFF}, kV	E^*_0, kV/cm
0.44	30	71	26.9	28.0	900
0.9	30	101	41.2	43.5	687
1.7	30	115	50.5	54.5	456
2.5	30	120	70.8	75.0	424

For a quasi-uniform field in a narrow gap the following approximate formula is valid for the electric field strength [2]

$$E \approx E_0/(1+(1-\cos\theta)/\beta)\ . \qquad (2)$$

The direction $\theta = 0$ corresponds to the apex of sphere. Only a small part of the electrode area near the symmetry axis makes a major contribution to breakdown inception because of the sharp dependence of the function $\mu(E)$ on the electric field. For this region, the approximate formula (2) practically coincides with the exact solution (1) up to $\beta = 0.02$ [2].

TABLE III.

β	0.01	0.05	0.066	0.1	0.2
$a(\beta)$	1.007	1.034	1.044	1.068	1.13

The maximum value of electric field on the electrode surface is somewhat higher than the average value E_0 (Fig. 2). The coordinate z along symmetry axis is expressed through bispherical coordinate ξ as $z = R\,\mathrm{sh}\xi_1\,\mathrm{sh}\xi/(\mathrm{ch}\xi+1)$. Use of correction factor $a(\beta) = E_{max}/E_0$ that depends only on the parameter β (Table III) allows one to extend the range of the applicability of (2) to the values $\beta \approx 0.1$, using $a(\beta)E_0$ instead of E_0.

MACROSCOPIC APPROACH

In stochastic approach offered in [1-4], macroscopic function $\mu(E)$ was introduced which depends on local electric field. It was supposed that probability of breakdown inception near small element of surface of electrode at time t does not depend on previous moments of time and on events near other elements of electrode. The function $\mu(E)$ has physical sense of probability density of breakdown initiation on a small element of electrode surface in a short interval of time. The probability of breakdown inception in time t is equal to

$$P_+(t) = 1 - \exp(-H) \tag{3}$$

where the value of integral of electric field action $H(t)$ is expressed through the function $\mu(E)$ and changes in time as

$$H(t) = \int_0^t \left(\int_S \mu(E)\,ds \right) dt . \tag{4}$$

For hemispherical electrodes with a small gap distance between them, it is possible to turn the integration in (4) from integral over the surface of electrode to the integral over electric field, using the approximation (2) [1, 2]

$$\int_S \mu(E)\,ds \approx d\pi R E_0 \int_0^{E_0} \frac{\mu(E)}{E^2} dE \quad . \tag{5}$$

On the right hand side of (5), we used zero as lower limit of integration, bearing in mind the extremely sharp dependence of $\mu(E)$ on electric field. Using (5), the concept of the effective area S_* for hemispherical electrodes was introduced [1]

$$S_* = \frac{d\pi R E_0}{\mu(E_0)} \int_0^{E_0} \frac{\mu(E)}{E^2} dE . \tag{6}$$

The effect of an increase in the area, on which the breakdown originated, with an increase in the product of electrode radius and gap spacing Rd is well known from the experiment [7]. The formula $S_* = d\pi Rk$ obtained from simple geometric relations is usually used in electrical engineering [8]. Here $k = \Delta E/(E_0 - \Delta E)$ where ΔE is the permissible field deviation from maximum field at apex of the sphere. Nevertheless, the relevant value of k is not well-defined and depends on the particular liquid. The value S_* introduced in our work (6) naturally depends on features of particular dielectric through the function $\mu(E)$ and also generally on electric field E_0.

For arbitrary function $\mu(E)$ for hemispherical electrodes

$$P_+ = 1 - \exp\left(-d\pi R \int_0^t \left(E_0(t) \int_0^{E_0(t)} \frac{\mu(E)}{E^2} dE \right) dt \right) . \tag{7}$$

If it is possible to approximate the function $\mu(E)$ for dielectric by the power-law dependence

$$\mu(E) = A(E/E_1)^n , \tag{8}$$

the analytical expressions for integral of electric field action $H(t)$ can be derived for AC voltage of slowly increasing amplitude $V = \sqrt{2}k_e t\sin(\omega t)$ for flat and hemispherical electrodes [3, 4]. For approximation (8) the value of effective area for hemispherical electrodes is $S_* = \pi Rd/(n-1)$.

The dependences of the probability of breakdown on such parameters as the radius of the electrode surface (or electrode area in the case of flat electrodes), gap distance, rate of increase in voltage, etc were derived. The value of electric strength depends only on parameter $b = k_e/(Sd)$ for flat electrodes. For hemispherical electrodes the effective area S_*

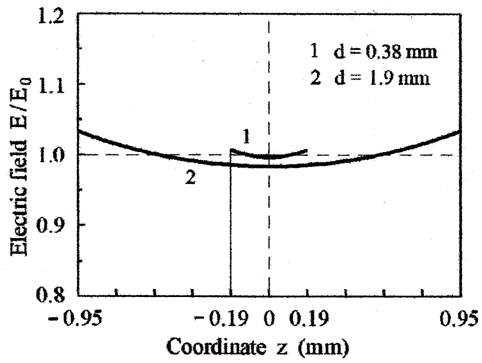

Fig. 2. Distributions of electric field strength along the symmetry axis of two spherical electrodes obtained from the exact solution (1) at $\beta = 0.01$ (curve *1*) and $\beta = 0.05$ (curve *2*). $R = 19$ mm.

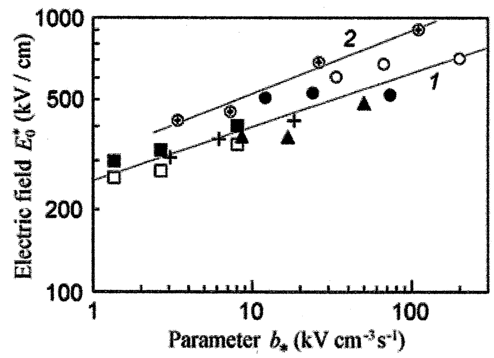

Fig. 3. Dependence E^*_0 on parameter b_*. Transformer oil (curve *1*) $d = 0.5$ (○), 0.83 (●), 1.0 (▲), 1.66 (+), 2.5 (□,■) mm. $R = 19$ mm. Perfluorodibutyl ether (curve 2, ⊕). $d = 0.44$, 0.9, 1.7, 2.5 mm. $R = 30$ mm.

should be used instead of S. Nevertheless, for hemispherical electrodes the parameter $b_* = k_e/(\pi Rd^2)$ is more convenient, which does not depend on n. The parameters b and b_* could be used for preliminary comparing of the experimental data obtained at different geometry of electrodes, different values of k_e, and S (for flat electrodes) or d and R (for hemispherical electrodes). For example, the results of the experiments (Tables I and II) were plotted as the dependences on the parameter b_* (Fig. 3).

One way to determine the function $\mu(E)$ from experiments is to use (7) with fixed values of H. It is convenient to use the values of electric field E^*_0 corresponding to $H = 1$, for which $P_+ = 0.63$ (the method of fixed probability).

However, power-law approximation (8) of function $\mu(E)$ gives too weak dependence on an electric field (Fig. 4, curve *1*) that results in too wide scattering of breakdown voltages in simulations in comparison with the experiments.

The approximation in the form

$$\mu = AE^2 \exp(E/g) \tag{9}$$

is sharper than approximation (8) and describes the histograms of breakdown voltages and breakdown pitting on a surface of hemispherical electrodes better. The approximation (9) also allows one to calculate the integral over electric field in (7)

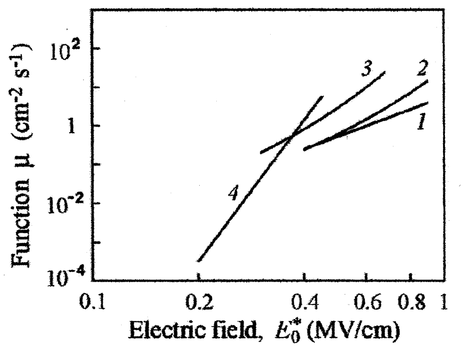

Fig. 4. Values of function $\mu(E)$ reconstructed from experiment. The power-law approximation of $\mu(E)$ for perfluorodibutyl ether (*1*). The approximations (9) of $\mu(E)$ for perfluorodibutyl ether (*2*) and transformer oil (*3*). Straight line *4* is the function $\mu(E)$ reconstructed in [4] for transformer oil from the data of [5].

analytically. Integration over time in (7) was carried out numerically right up to the moment corresponding to amplitude value of electric field E^*_0. Then, the parameters A and g were obtained using the condition $H = 1$ for each series of breakdowns. The values of function $\mu(E)$ reconstructed from data of present work are shown in Fig. 4 (curves *2* and *3*).

STOCHASTIC SIMULATION

Using the reconstructed function $\mu(E)$, one can plot any dependences of the breakdown initiation probability for various geometry of electrodes and also for various magnitude, duration, and waveform of the applied voltage.

Within the framework of the stochastic approach proposed, computer simulation of series of breakdowns was carried out using (3), (7), and (9). Statistical time lag before breakdown t_S was determined from

$$\int_0^{\omega t_S} \tau|\sin\tau|\left[e^{B\tau|\sin\tau|}-1\right]d\tau = -\frac{\omega^2}{\sqrt{2\pi}\,Agk_e R}\ln(\zeta), \qquad (10)$$

where $B = \sqrt{2}k_e/(\omega d g)$, ζ is a random number that is uniformly distributed in the interval from 0 to 1. The integration on the left hand of (10) was carried out numerically until the value of the integral was equal to the value of expression on the right hand. Typical series of breakdowns in transformer oil obtained in computer simulations is shown in Fig. 1,*b*. The results are in good agreement with the experimental ones.

CONCLUSIONS

Probability distributions of breakdown initiation were analytically obtained for flat, hemispherical and coaxial cylindrical electrodes under DC voltage, ramp voltage and AC voltage of linearly increasing amplitude. Usually the experimental data are fitted in log-log scale by Weibull functions (on electric field, area of electrodes and time) [9-11]. The probability distribution (3) with our definition of function $H(t)$ in form (4) and (5) is more general. Using power-law approximation (8) for $\mu(E)$, it is possible to obtain Weibull-like distributions for flat and hemispherical electrodes that depend on two parameters A and n in (8). Hence, the Weibull distributions are in some sense close to the particular forms of our distribution. The direct computer simulations of experiments on breakdown in dielectric liquids describe essentially stochastic nature of breakdown that is necessary to take into account at designing high power electrical apparatus.

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