# Probability Density Function of Electrical Breakdown Initiation in Dielectric Liquids under AC and DC Voltage

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***Abstract*** *- The stochastic regularities of breakdown initiation in dielectric liquids were studied using the theory proposed in [1,2]. The electric strength of perfluorodibutyl ether was investigated experimentally at AC voltage of linearly increasing amplitude. Analytical expressions were derived that allowed us to reconstruct the parameters of the probability function* $\mu(E)$ *for perfluorodibutyl ether, n-hexane and transformer oil using the experimental data of present work, [3], and [4], respectively.*

## 1 Introduction

It is well known that the prebreakdown processes in liquid dielectrics have a stochastic nature. Thus, a correct description of dielectric breakdown has to include probability distribution functions for such processes. One of them is the initiation of breakdown due to the development of a series of microscopic phenomena at the electrode surface and in a thin dielectric layer contiguous to it. The duration of this stage of breakdown (called the statistical time lag $t_S$) is a random value for which the probability density depends on the electric field and its distribution along the surface of the electrodes.

A macroscopic function $\mu(E)$ introduced in **[**1,2**]** depends on the local electric field and is the probability density of the breakdown initiation from a small element of the electrode area in a short time interval. The macroscopic approach allows one to reconstruct the function $\mu(E)$ from experimental data and then use it for modeling of breakdown.

## 2 Experiments

Perfluorocarbon liquids are well known as good dielectrics, which have high resistivity and electrical strength as transformer oil and close values of permittivity of about 2. In addition, they have very low viscosity and high density and are not flammable, which makes these liquids very promising for industrial applications.

The experiments were conducted with the perfluorodibutyl ether $CF_3(CF_2)_3–O–(CF_2)_3CF_3$. The liquid was previously filtered and boiled for degassing over a period of 1 to 2 hours at the temperature 101 $^oC$ with a returning cooler to prevent its boiling out. Hemispherical stainless steel and brass electrodes with surface radii $R$ = 30 and 40 mm, respectively, were used. Before each series of experiments, the surfaces of the electrodes were polished.

Usually, the time lag of breakdown measured in experiments includes not only the statistical time lag $t_S$ but also the formation time $t_F$ . To obtain data on the statistical time lag, it is necessary to register the moments of streamer inception. However, it is more convenient to use a small gaps, in which $t_F << t_S$ [1].

The effective value of AC voltage increased with a constant rate $k$ = 2 kV/s. The frequency was 50 Hz. A special electronic device was used to remove the voltage from the electrodes immediately after breakdown.

## 3 Results

In all series of breakdowns, one can see rather significant statistical difference in breakdown voltages. This is a direct consequence of the stochastic regularities of the process. A typical series of breakdowns is shown in Fig. 1.

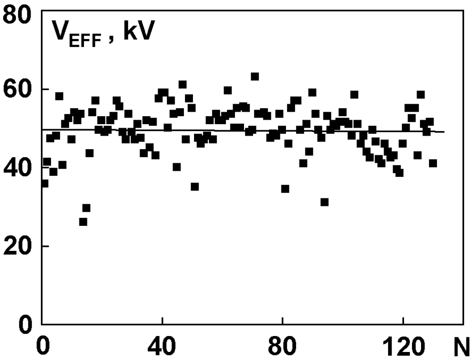

**Fig. 1.** Typical series of breakdowns in perfluorodibutyl ether for brass electrodes of radius $R$ = 40 mm at gap distance $d$ = 1.7 mm.

Results of the experiments are presented in Table 1. Here $d$ is the gap length, $N_0$ is the number of breakdowns in series, $V^*_{EFF}$ is the effective voltage in each series of experiments at which breakdown did not yet occur with some fixed probability $P_-$ = 0.37, $E^*_0$ is the corresponding amplitude value of the average electric field along the axis between the electrodes.

The measured electrical strength of perfluorodibutyl ether is in reasonable agreement with the values of the order of 200 kV/cm measured for similar perfluorocarbon liquids in [4,5] for electrode areas significantly larger than those in the present work. A more detailed analysis of the results obtained can be performed only using a theoretical model.

Table 1.

| $d$, mm | $R$, mm | $N_0$ | $\langle V_{\mathrm{EFF}}\rangle$, kV | $V^*_{\mathrm{EFF}}$, kV | $E^*_0$, kV/cm |
|---|---|---|---|---|---|
| Brass electrodes | | | | | |
| 0.44 | 40 | 210 | 18.7 | 21.0 | 477 |
| 0.9 | 40 | 135 | 37.7 | 42.0 | 469 |
| 1.7 | 40 | 130 | 49.4 | 52.0 | 307 |
| 2.5 | 40 | 80 | 73.4 | 77.0 | 308 |
| Stainless steel electrodes | | | | | |
| 0.44 | 30 | 140 | 23.3 | 26.0 | 591 |
| 0.9 | 30 | 101 | 41.2 | 43.5 | 486 |
| 1.7 | 30 | 115 | 50.5 | 54.5 | 322 |
| 2.5 | 30 | 120 | 70.8 | 75.0 | 300 |

## 4 Theory

The stochastic regularities of breakdown in liquids were studied using the theory of breakdown initiation developed in [1,2]. In this theory, a new approach was developed. It was proposed to use a macroscopic function μ(*E*) to describe the initiation of a breakdown. This function is the probability density of the origin of a streamer. The parameters of the function μ(*E*) depend on the properties of a specific dielectric and perhaps on the material of the electrodes. This macroscopic approach allows one to obtain the dependencies of the breakdown initiation probability in time on the applied voltage, its waveform, electrode area, and gap length. And vice versa, it is possible to reconstruct the function μ(*E*) from experimental data. In the simplest case of DC voltage and flat electrodes

$$\mu(E) = \left(\langle t_S\rangle \cdot S\right)^{-1}\ , \qquad (1)$$

where *S* is the area of the electrode.

Only a small part of the hemispherical electrode area near the symmetry axis makes a major contribution to breakdown inception because of the sharp dependence of the function μ(*E*) on the electric field. For a narrow gap, the following approximate formula is valid [1,2]:

$$E \approx \frac{E_0}{1+(1-\cos\theta)/\beta}\ . \qquad (2)$$

Here $E_0 = V/d$ is the average electric field along the axis between the electrodes, $\beta = d/2R$, and $\theta$ is the polar angle at the sphere from the symmetry axis. For this region, the approximate formula (2) practically coincides with the exact solution obtained by solving the Laplace equation in bispherical coordinates in the interelectrode gap. For example, the difference between them is less than 2% of the maximum field strength for β = 0.02 [2].

Using the approximation (2) for narrow gaps, the following expression for the mean value of the statistical time lag was obtained in [1] for a stepwise voltage pulse:

$$\frac{1}{\langle t_S\rangle} = \int_S \mu(E)\,ds \approx d\pi R E_0 \int_0^{E_0} \frac{\mu(E)}{E^2}\,dE\ . \qquad (3)$$

Thus, the mean statistical time lag of breakdown for hemispherical electrodes is shown to be inversely proportional to the product of electrode radius and gap length.

## 5 Theory of breakdown at AC voltage

In [6] it is shown that the breakdown does not occur during time interval *t* with the probability $P_{-}(t) = \exp(-H)$ when the applied voltage depends on time. The value

$$H(t) = \int_0^t \left( \int_S \mu(E)\,ds \right) dt$$

has the physical meaning of a dimensionless analogue of the statistical time lag and can be called the field action integral. The probability of dielectric breakdown occurring during time interval *t* is $P_{+}(t) = 1 - \exp(-H)$, respectively.

In the case of AC voltage with linearly increasing amplitude, $V = kt\sin(\omega t)$. Using the power approximation $\mu(E) = AE^n$ for the function μ(*E*) for flat electrodes of area *S* with the gap distance *d*, one obtains the expression

$$H(t) = C \int_0^{\omega t} z^n\,|\sin(z)|^n\,dz\ ,$$

where $C = \dfrac{ASk^n}{d^n\omega^{n+1}}$. The plot of $P_+$ versus the dimensionless time *t*/*T* is shown in Fig. 2.

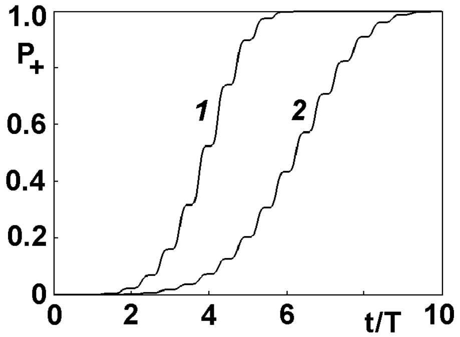

**Fig. 2.** The probability of breakdown initiation for flat electrodes vs. *t*/*T* , where *T* is the period of AC voltage.

At AC voltage of a slowly increasing amplitude, the product *kt* changes only slightly during each half-cycle and the voltage pulse form is practically proportional to $\sin(\omega t)$. In this case, the action integral changes over a half-cycle by the value

$$\Delta H_i = \frac{SAE_i^n T}{2\pi} \int_0^{\pi} \sin^n(z)\,dz\ ,$$

where $E_i$ is the amplitude of the electric field when the number of voltage half-cycles is equal to $i$.

When breakdown occurs after many voltage half-cycles, we have

$$H_i = \sum_{j=1}^{i} \Delta H_j \approx \frac{SdAE_i^{n+1}}{k\pi(n+1)} \int_0^{\pi} \sin^n(z)\,dz \cdot \qquad (4)$$

From this expression, we obtain a formula that expresses the values of $\mu(E)$ in terms of the experimental distribution of breakdown voltages:

$$\mu(E) = \frac{\pi k \ln(N_i / N_{i+l})}{Sd\,\Delta E_l \int_0^{\pi} \sin^n(\omega t)\,dt} \cdot \qquad (5)$$

Here $N_i$ and $N_{i+1}$ are the numbers of breakdowns in the series that occurred not earlier than the $i$ and ($i$+$l$) voltage half-cycles, respectively, after the voltage was switched on, $\Delta E_l$ is the increment of the electric-field strength over $l$ half-cycles (it was assumed that $l << i$ ).

Using the approximations (2) and (3) we obtain the following approximate formula for hemispherical electrodes with a small interelectrode gap:

$$\Delta H_i = \frac{Rd\,AE_{i0}^{n}\,T}{2(n-1)} \int_0^{\pi} \sin^n(z)\,dz \cdot$$

Here $E_{i0}$ is the amplitude of the average electric field on the axis between the electrodes. By analogy,

$$H_i = \frac{Rd^2 AE_{i0}^{n+1}}{k(n^2-1)} \int_0^{\pi} \sin^n(z)\,dz \cdot \qquad (6)$$

Thus, we have the dependencies of the probability of breakdown on the main parameters such as the radius of the electrode surface (or electrode area in the case of flat electrodes), gap distance, rate of increase in voltage, etc.

To analyze the experimental data, it is convenient to take the value of electric field $E^*_0$ corresponding the value $H$ = 1, for which the probability of absence of breakdown $P_{-}$ = 0.37.

## 6 Reconstruction of μ(E) from experimental data

The analytical expressions (3), (4), (5) and (6) derived in the present work allow one to reconstruct the function $\mu(E)$ from experimental data and to obtain the parameters $A$ and $n$ of the power-law approximation for $\mu(E)$.

The approach proposed was tested using the experimental data of [3] on breakdown in n-hexane for narrow gaps. These experiments were carried out for DC voltage pulses in the range of $E_0$ from 1 to 3.5 MV/cm at a pressure of $P_0$ = $10^5$ Pa. Hemispherical electrodes made of stainless steel with surface radius $R$ = 0.5 cm were used. Expression (3) is in good agreement with the experimental data if the approximation

$$\mu(E) = A(E/E_1)^n \quad \text{with} \quad E_1 = 1\ \text{MV/cm} \qquad (7)$$

is used, where $n$ = 4.65, $A$ = 9.4·10$^7$ cm$^{-2}$·c$^{-1}$ [2].

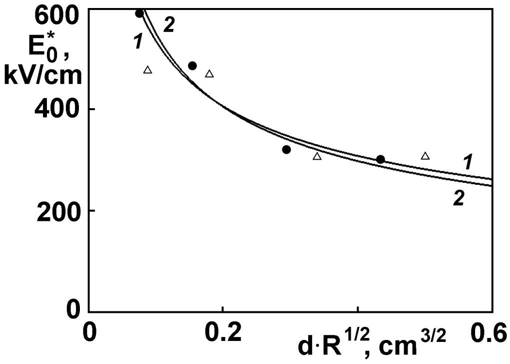

**Fig. 3**. The dependence of amplitude of average electric field of breakdown $E^*_0$ ($P_{-}$= 0.37) on value $d\sqrt{R}$ for electrodes of brass (Δ) and stainless steel (•).

Experimental data on breakdown of perfluorodibutyl ether are shown in Fig. 3 as a dependence on value $d\sqrt{R}$ . The fittings (6) are reasonable if the approximation (7) is used, where $n$ = 4 and $A$ = 57.4 cm$^{-2}$·c$^{-1}$ for brass electrodes (curve *1*) and $n$ = 3.5, $A$ = 26.2 cm$^{-2}$·c$^{-1}$ for stainless steel electrodes (curve *2*). Practically we have no difference between results obtained with stainless steel and brass electrodes (Fig. 3).

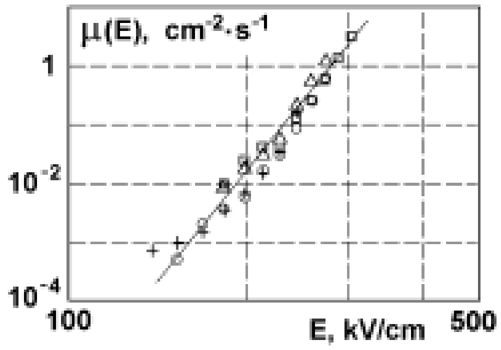

**Fig. 4.** Reconstructed values of μ($E$) for transformer oil. $S$ = 2.45( ◻), 8.33(Δ), 23.5(o), 49(+) cm$^2$.

The experimental data of [7] on breakdown in transformer oil under AC voltage with a frequency of 60 Hz were also analyzed using this approach. The effective value of the applied voltage increased with a constant rate $k$ = 3 kV/s. In these experiments, four pairs of flat electrodes with areas of 2.45, 8.33, 23.5, and 49 cm$^2$ were used at a gap spacing $d$ = 0.19 cm. The results of reconstruction of the function $\mu(E)$ obtained from (5) for each pair of electrodes are shown in Fig. 4. Fitting of these results with the least-square method gives the values of parameters in (7) $n$ = 12.3 and $A$ = 5.0·10$^6$ cm$^{-2}$·c$^{-1}$ (the straight line in Fig. 4).

## 7 Calculations

Using the reconstructed function $\mu(E)$, one can plot any dependencies of the breakdown initiation probability for various electrode geometry and also for various magnitude, duration, and waveform of the applied voltage.

For example, we carried out the simulations of breakdown pitting on the surface of hemispherical electrodes (Fig. 5,*a*). It is in reasonable agreement with experimental results (Fig. 5,*b*).

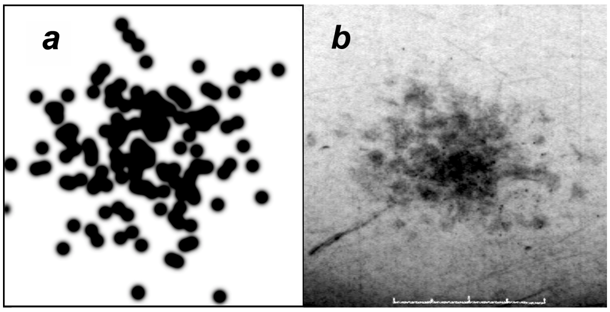

**Fig. 5.** Distribution of breakdown pitting on the electrode surface. (*a*) computer simulation of series of breakdown in perfluorodibutyl ether. (*b*) the experiments on breakdowns in perfluorodibutyl ether with stainless steel electrodes. In both pictures the areas 8×8 mm are shown. $R = 30$ mm, $d = 0.44$ mm, $N_0 = 140$.

Using the expression (4) and the function $\mu(E)$ reconstructed for transformer oil, we calculated the field action integrals $H_i$ for each of 1600 breakdowns in [7]. Figure 6 shows the distribution of these values of $H_i$. One can see that the bar chart agrees well with Poisson's probability distribution $N = N_0 \exp(-H)$ (curve *1*). The values $-\ln(N/N_0)$ versus $H$ are closed to the straight line *2*, that also confirms the theory proposed.

## 8 Discussion and conclusions

The function $\mu(E)$ was introduced to obtain an analytical explanation of experimental dependencies. Such formulae are obtained in the case of flat and hemispherical electrodes at DC voltage and AC voltage of linearly increasing amplitude.

Within the framework of the theory proposed, the effect of enhancement of the electric strength of liquid dielectrics with decrease in the gap length between hemispherical electrodes was explained analytically [1,2]. As the gap length decreases, the electrical strength of dielectric increases mainly due to a decrease in the geometrical region in which the electric field is close to $E_0$ in accordance to the approximate formula (3). The second reason is that the maximum electric field at the electrode surface $E_{\mathrm{MAX}}$ (at $\theta = 0$) is greater than $E_0$ and tends to $E_0$ as $d$ decreases. Hence, the value of $\mu_{\mathrm{MAX}}$ decreases with decrease in the gap distance even if the average value at the axis $E_0 = \text{const}$.

The theory of breakdown initiation offered here and the values of the functions $\mu(E)$ measured for n-hexane, perfluorodibutyl ether, and transformer oil can be useful for constructing electrotechnical devices in which liquid dielectrics are employed.

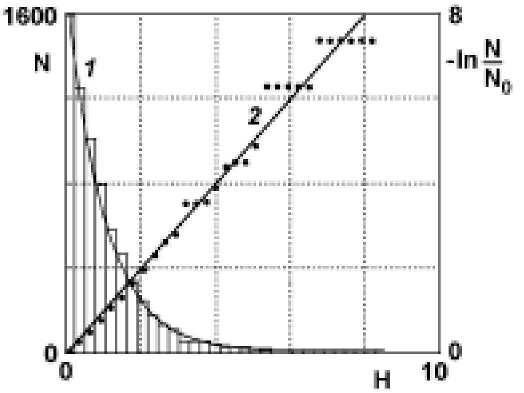

**Fig. 6.** Bar chart of the distribution of *H*-values corresponding to the experimental data [7] on breakdowns in transformer oil. $N_0 = 1600$.

## 8 Acknowledgements

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