ANISOTROPIC ELECTROHYDRODYNAMIC INSTABILITY DUE TO ACTION OF ELECTROSTRICTIVE FORCES

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A homogeneous state below the coexistence curve can be either metastable (if $\partial p/\partial \rho > 0$) which can exist for relatively long time and finally decays into a two-phase system of pure liquid and vapor through the process of nucleation [1, 2], or unstable ($\partial p/\partial \rho < 0$). Unstable states are thermodynamically prohibited, a homogeneous matter decays very fast through a spinodal decomposition [3]. Experiments [4] revealed that electric field influences the region of liquid stability.

The body force acting on dielectric liquid is given by the Helmholtz formula [5]

$$\mathbf{F} = q\mathbf{E} - \frac{E^2}{8\pi}\nabla\varepsilon + \frac{1}{8\pi}\nabla\left[E^2\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right]. \tag{1}$$

For gases and liquids with weakly polarizable molecules, the permittivity depends linearly on fluid density

$$\varepsilon = 1 + 3\alpha\rho\,, \tag{2}$$

where $\alpha = 4\pi\beta/(3m)$, β is the molecular polarizability, m is the mass of a molecule.

For nonpolar liquids, the Clausius – Mosotti law [6] is valid

$$\varepsilon = 1 + 3\alpha\rho/(1-\alpha\rho)\,. \tag{3}$$

For polar dielectrics, the Onsager – Kirkwood – Fröhlich law [6] is valid, but it is more reliable to use the experimental values of ε, $(\partial\varepsilon/\partial\rho)_T$, $(\partial^2\varepsilon/\partial\rho^2)_T$.

In present work, a linear stability analysis of the Euler equations for dielectric liquid under the action of electric field in the simplest isothermal case was carried out. A constant voltage was applied between two plane horizontal electrodes (only the vertical component of electric field E_z was non-zero). Let us consider a growth of small one-dimensional harmonic perturbations of density and velocity corresponding to the stratification of matter along the electric field

$$\rho = \rho_0 + A_0\exp(\gamma t)\exp(i2\pi x/\lambda)\,, \qquad u_x = C_0\exp(\gamma t)\exp(i2\pi x/\lambda) \tag{4}$$

and to the stratification across the electric field

$$\rho = \rho_0 + A_0\exp(\gamma t)\exp(i2\pi z/\lambda)\,, \qquad u_z = C_0\exp(\gamma t)\exp(i2\pi z/\lambda)\,, \tag{5}$$

where λ is the wave length, A_0, C_0 are the initial amplitudes of perturbations, γ is the instability increment, ρ_0 is the mean density of a matter.

The body force in the case of the perturbation (4) has the form

$$F_x = \frac{E_0^2\rho}{8\pi}\left(\frac{\partial^2\varepsilon}{\partial\rho^2}\right)_T\frac{\partial\rho}{\partial x} = K_x\frac{\partial\rho}{\partial x}\,, \tag{6}$$

where E_0 is the (constant) magnitude of uniform electric field. For the perturbation (5), the magnitude of electric displacement D_0 is constant in space and we have

$$F_z = \frac{D_0^2 \rho}{8\pi\varepsilon^2}\left(\left(\frac{\partial^2 \varepsilon}{\partial \rho^2}\right)_T - \frac{2}{\varepsilon}\left(\frac{\partial \varepsilon}{\partial \rho}\right)_T^2\right)\frac{\partial \rho}{\partial z} = K_z \frac{\partial \rho}{\partial z}. \quad (7)$$

In both cases, the instability increment is given by

$$\gamma = 2\pi\left(\sqrt{-(\partial p / \partial \rho)_T + K}\right)/\lambda. \quad (8)$$

Since always $K_z < K_x$, the equation of spinodal curve has the form

$$(\partial p/\partial \rho)_T = K_x. \quad (9)$$

The instability increment (8) tends to infinity as wave length tends to zero. However, the linear stability analysis of the one-dimensional Navier – Stoke equations for a viscous fluid gives the following value of instability increment

$$\gamma = 2\pi\left(\sqrt{(\pi b/\lambda)^2 + K - (\partial p / \partial \rho)_T} - \pi b/\lambda\right)/\lambda, \quad (10)$$

where $b = (4/3\mu + \xi)/\rho_0$, μ and ξ are the dynamic and the second viscosities. For viscous fluid, the instability increment is almost constant $\gamma_{\max} \approx (K - (\partial p/\partial \rho)_T)/b$ for $\lambda < \lambda_* \sim 2\pi b/\sqrt{K - (\partial p / \partial \rho)_T}$. Instability increment is limited by viscous forces. The boundary of instability on $\tilde{T} - \tilde{\rho}$ diagram is the same as obtained from (9).

The values for K are different for stratifications along and across the field. For both the polar and nonpolar dielectric liquids $K_x > 0$, hence electric field increases the instability increment for perturbation of type (4). In all cases considered, $K_z < 0$, hence, the stability of a matter with respect to the stratification across the field is increased. Thus, for $K_x > (\partial p / \partial \rho)_T$ the anisotropic decay of homogeneous fluid into system of vapor filaments in a liquid parallel to the field occurs.

For the “gas” law (2) $K_x = 0$, hence, the instability is possible only in the region of forbidden states $(\partial p/\partial \rho)_T < 0$ same as in the case without electric field. Even in this case, the instability is anisotropic because $K_z = -D_0^2(\varepsilon - 1)^2/(4\pi\varepsilon^3\rho) < 0$.

In an electric field, the critical point is shifted both in temperature and density [5]. The equation of the spinodal curve $(\partial p/\partial \rho)_T = E_0^2 \rho(\partial^2 \varepsilon / \partial \rho^2)_T / 8\pi$ follows immediately from equations (6) and (9). This boundary of hydrodynamic stability exactly coincides with the boundary of thermodynamic stability of dielectric liquids obtained in [5]. However, the possibility of anisotropic instability and, consequently, the possible stratification of a matter were not considered in [5].

As an example, we used the van der Waals equation of state in reduced variables

$$\tilde{p} = 8\tilde{T}\tilde{\rho}/(3 - \tilde{\rho}) - 3\tilde{\rho}^2. \quad (11)$$

For nonpolar liquids (3), we have $K_x = E_0^2(\varepsilon - 1)^2(\varepsilon + 2)/(36\pi\rho)$. In this case, the formula of the spinodal curve can be written in an explicit form

$$\tilde{T} = \frac{\tilde{\rho}(3 - \tilde{\rho})^2}{4}\left(1 + \tilde{A}\frac{(\alpha\rho_{cr})^2}{(1 - \alpha\rho_{cr}\tilde{\rho})^3}\right). \quad (12)$$

The second term in parentheses corresponds to the shift of the critical point in temperature, and $\tilde{A} = E_0^2/(8\pi p_{cr})$. For parameters corresponding to argon $T_{cr} = 151$ K, $\rho_{cr} = 531$ kg/m^3, $p_{cr} = 4.86$ MPa, $\alpha\rho_{cr} = 0.057$, the spinodal curves were calculated and are shown in Fig. 1 for $E = 0$ and for $\tilde{A} = 100$. The shift of the critical point in density $\Delta\tilde{\rho}_{cr} \sim 2(\alpha\rho_{cr})\Delta\tilde{T}$ is very small because $\alpha\rho_{cr} << 1$.

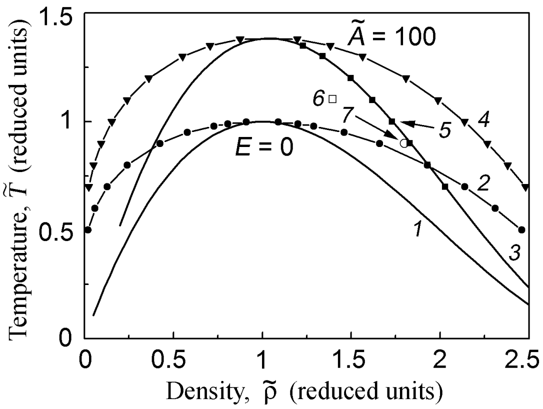

Fig. 1. Coexistence curves (*2* and *4*) and theoretical spinodals (12) (curves *1* and *3*) for the van der Waals equation of state (11). Curves *1, 2* – without electric field, *3, 4* – in uniform electric field at $\tilde{A} = 100$. *5* – points of the spinodal obtained in hydrodynamic simulations. Points *6* and *7* are the states initially above the critical point ($\tilde{\rho}_0 = 1.4$, $\tilde{T} = 1.1$) and in the region of stability of liquid phase ($\tilde{\rho}_0 = 1.8$, $\tilde{T} = 0.9$), respectively.

We performed the simulations of the evolution of homogeneous dielectric fluid that was initially at rest in uniform electric field (initial random density perturbations were $\Delta\rho/\rho_0 \sim 10^{-6}$). The dynamics of continuous media was simulated using the lattice Boltzmann equation (LBE) method [7, 8] modified to electrohydrodynamic problems with possible phase transition [9, 10]. Periodic boundary conditions in *x* direction were used. The neutral wetting of electrodes was assumed (wetting angle equal to $\pi/2$). The distribution of electric field was obtained by solving the equations $\text{div}(\varepsilon\nabla\varphi) = 0$ and $\mathbf{E} = -\nabla\varphi$ with corresponding boundary conditions $\varphi = 0$ and $\varphi = E_0 L_y$ at the lower and upper electrodes. The simulations were performed on a 150×150 lattice.

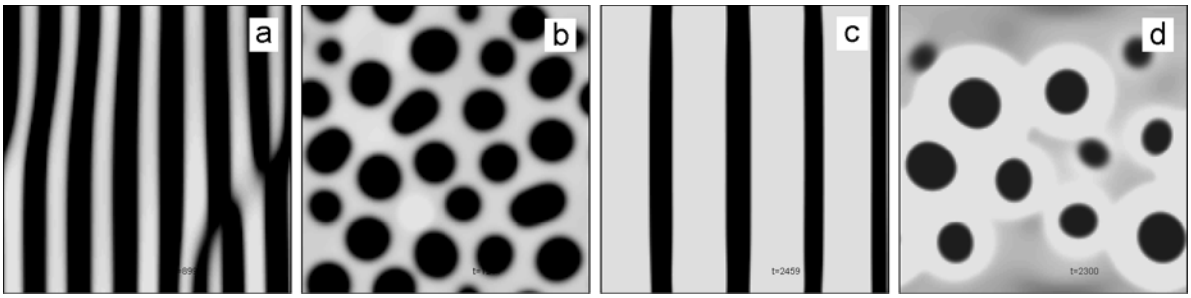

Fig. 2. Anisotropic stratification of fluid along the initially vertical electric field (*a*, *c*). Development of instability in the plane *x–y* perpendicular to the field (*b*, *d*). The lower density is shown by dark color. (*a*, *b*) – state *6* in Fig. 1; (*c*, *d*) – state *7* in Fig. 1. $\tilde{A} = 100$. Lattice 150×150.

For nonpolar dielectric (3) the coexistence (binodal) curves were obtained in computer simulations both without electric field (Fig. 1, curve *2*) and in an initially vertical uniform electric field (curve *4*). The high-density part of the spinodal curve (points *5*) was also calculated. The diagram obtained shows that the anisotropic decay of liquid along the sufficiently high electric field is possible for matter being initially in metastable and even in stable states (for example, states *6* and *7* in Fig. 1)

This stratification along a uniform electric field was indeed observed in computer simulations for matter that was initially both in a state above the critical point (Fig. 2,*a*,*b*) and in a stable liquid state (Fig. 2,*c*,*d*). Instability arose in form of channels of approximately circular cross-section that generated compression waves during expansion (Fig. 2,*d*). This is a cooperative effect in theory of nucleation [2].

In all previous works ([11, 12] and others), only the possibility of generation of spherical or ellipsoidal vapor bubbles was considered. The anisotropic instabilities were not considered at all. The mechanism of streamer growth in form of a crack in a liquid containing population of initial sub-microscopic spherical holes [11] is fundamentally different from the mechanism of anisotropic instability.

In the process of breakdown of liquid dielectrics in strong electric fields that can locally reach the values of ~1–100 MV/cm (for different liquids), the proposed anisotropic instability is possibly the key mechanism of inception of streamer structures and their ultra-fast propagation in a form of thin filaments (the velocity can exceed 100 km/s [13]) oriented on average along the local electric field (Fig. 2,*a*,*c*). Since the electric strength of vapor is relatively low, an electric breakdown can occur in some of vapor channels produced by the anisotropic instability. After a filament becomes conductive, the electric field ahead of this filament is enhanced. The electric field in neighbor non-conductive channels decreases, and these channels disappear if their states leave the region of instability. This process can propagate very fast step by step in a space between electrodes. This work was supported by the Russian Foundation for Basic Researches (grant N 06-08-01006-a).

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